

**PRACTICAL ANALYSIS OF  
COMPOSITE STRUCTURES**

**J. N. Reddy and A. Miravete**

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# **PRACTICAL ANALYSIS OF COMPOSITE STRUCTURES**

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## PREFACE

Composite materials are increasingly used in aerospace, under water, and automotive structures. The application of composite materials to engineering components has spurred a major effort to analyze structural components made from them. Composite materials provide unique advantages over their metallic counterparts, but they also present complex and challenging problems to analysts and designers.

The global deformation of laminated composite structures is often characterized by complex coupling between the extension, bending, and shearing modes. Further, due to their low transverse shear stiffness, composite laminates often exhibit significant transverse shear deformation at lower thickness-to-span ratios than do similar homogeneous isotropic plates and shells. To take advantage of the full potential of composite materials, structural analysts and designers must have accurate analysis tools and design methods at their disposal.

The primary aim of this book is to present a summary of the equations governing composite laminates and show a way to analyze most common types of composite structural elements from a practical point of view. A number of experimental results for several types of structures are included, and theoretical and experimental correlations are discussed. A PC based software, *DAC* (Designing Advanced Composites), is discussed in the last chapter. The book serves as a reference for practicing engineers and designers of composite materials. It can also be used as a text book at the graduate level.

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