An Introduction to Nonlinear Finite Element Analysis

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To

My beloved teacher

Professor John Tinsley Oden

About the Author

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Dr. Reddy is internationally known for his contributions to theoretical and applied mechanics and computational mechanics. He is the author of over 300 journal papers and dozen other books, including *Introduction to the Finite Element Method* (Second Edition), McGraw-Hill, l993; *Theory and Analysis of Elastic Plates*, Taylor & Francis, 1999; *Energy Principles and Variational Methods in Applied Mechanics* (Second Edition), John Wiley, 2002; *Mechanics of Laminated Plates and Shells: Theory and Analysis,* (second Edition) CRC Press, 2004; *An Introduction to the Mathematical Theory of Finite Elements* (coauthored with J. T. Oden), John Wiley, l976; *Variational Methods in Theoretical Mechanics* (coauthored with J. T. Oden), Springer–Verlag, 1976; *The Finite Element Method in Heat Transfer and Fluid Dynamics* (Second Edition; coauthored with D. K. Gartling), CRC Press, 2001; and

Professor Reddy is the recipient of the *Walter L. Huber Civil Engineering Research Prize* of the American Society of Civil Engineers (ASCE), the *Worcester Reed Warner Medal* and the *Charles Russ Richards Memorial Award* of the American Society of Mechanical Engineers (ASME), the 1997 *Archie Higdon Distinguished Educator Award* from the American Society of Engineering Education (ASEE), the 1998 *Nathan M. Newmark Medal* from the American Society of Civil Engineers, the 2000 *Excellence in the Field of Composites* from the American Society of Composites, the 2000 *Faculty Distinguished Achievement Award for Research* and the 2003 *Bush Excellence Award for Faculty in International Research* from Texas A&M University, and the 2003 *Computational Solid Mechanics Award* from the U.S. Association of Computational Mechanics (USACM).

Dr. Reddy is a Fellow of ASCE, ASME, the American Academy of Mechanics (AAM), the American Society of Composites, the U.S. Association of Computational Mechanics, the International Association of Computational Mechanics (IACM), and the Aeronautical Society of India (ASI). Professor Reddy is the Editor-in-Chief of *Mechanics of Advanced Materials and Structures*, *International Journal of Computational Engineering Science* and *International Journal of Structural Stability and Dynamics*, and he serves on the editorial boards of over two dozen other journals, including *International Journal for Numerical Methods in Engineering, Computer Methods in Applied Mechanics and Engineering,* and *International Journal of Non-Linear Mechanics*.

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Preface

The objective of this book is to present the theory and computer implementation of the finite element method as applied to simple nonlinear problems of heat transfer and similar field problems, fluid mechanics, and solid mechanics. Both geometric as well as material nonlinearities are considered, and static and transient (i.e. time-dependent) responses are studied. The guiding principle in writing the book was to make the presentation suitable for (a) adoption as a text book for a first course on nonlinear finite element analysis (or for a second course following an introductory course on the finite element method), and (b) for use by engineers and scientists from various disciplines for self study and practice.

There exist a number of books on nonlinear finite elements. Most of these books contain a good coverage of the topics of structural mechanics, and few address topics of fluid dynamics and heat transfer. While these books serve as good references to engineers or scientists who are already familiar with the subject but wish to learn advanced topics or latest developments, they are not suitable as textbooks for a first course or for self study on nonlinear finite element analysis.

The motivation and encouragement that led to the writing of the present book have come from the users of the author's book, *An Introduction to the Finite Element Method* (McGraw–Hill, 1984; Second Edition, 1993; third edition scheduled for 2004), who have found the approach presented there to be most suitable for any one – irrespective of their scientific background – interested in learning the method, and also from the fact that there does not exist a book that is suitable as a textbook for a first course on nonlinear finite element analysis. The author has taught a course on nonlinear finite element analysis many times during the last twenty years, and the present book is an outcome of the lecture notes developed during this period. The same approach as that used in the aforementioned book, namely, the *differential equation approach*, is adopted in the present book to introduce the theory, formulation, and computer implementation of the finite element method as applied to nonlinear problems of science and engineering.

Beginning with a model (i.e. typical) second-order, nonlinear differential equation in one dimension, the book takes the reader through increasingly complex problems of nonlinear beam bending, nonlinear field problems in two dimensions, nonlinear plate bending, nonlinear formulations of solid continua, flows of viscous incompressible fluids in two dimensions (i.e. Navier–Stokes equations), time-approximation schemes, continuum formulations of shells, and material nonlinear problems of solid mechanics.

As stated earlier, the book is suitable as a textbook for a first course on nonlinear finite elements in civil, aerospace, mechanical, and mechanics departments as well as in applied sciences. It can be used as a reference by engineers and scientists working in industry, government laboratories and academia. Introductory courses on the finite element method, continuum mechanics, and numerical analysis should prove to be helpful.

The author has benefited in writing the book by the encouragement and support of many colleagues around the world who have used his book, *An Introduction to the Finite Element Method*, and students who have challenged him to explain and implement complicated concepts and formulations in simple ways. While it is not possible to name all of them, the author expresses his sincere appreciation. In particular, it is a pleasure to acknowledge the help of the author's students Juan P. Pontaza with the least-squares finite element analysis of fluid flow problems in Chapters 7 and 8, and Goy Teck Lim with the plasticity example in Chapter 10. The author expresses his deep sense of gratitude to his teacher, Professor J. T. Oden (University of Texas at Austin), to whom this book is dedicated and without whose advice, mentorship and support it would not have been possible for the author to modestly contribute to the field of applied mechanics in general and theory and application of the finite element method in particular, through author's teaching, research, and technical writings.

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