SOLUTION TO TUTORIAL/QUIZ 2

Exercise Problem 1

Assuming that Young's modulus varies with z, (i.e., $E = E(z)$), derive the governing equations of the Euler-Bernoulli beam in terms of the displacements u and w.

$$N_{xx} = \int_A \sigma_{xx}\, dA = A_{xx}\frac{\partial u}{\partial x} + B_{xx}\frac{\partial \theta_x}{\partial x}$$

$$M_{xx} = \int_A z\sigma_{xx}\, dA = B_{xx}\frac{\partial u}{\partial x} + D_{xx}\frac{\partial \theta_x}{\partial x}$$

$$(A_{xx}, B_{xx}, D_{xx}) = \int_A (1, z, z^2) E(z)\, dA$$

$$\frac{\partial}{\partial x}\left(A_{xx}\frac{\partial u}{\partial x} + B_{xx}\frac{\partial \theta_x}{\partial x}\right) + f = 0$$

$$\frac{\partial^2}{\partial x^2}\left(B_{xx}\frac{\partial u}{\partial x} + D_{xx}\frac{\partial \theta_x}{\partial x}\right) + q = 0$$

Exercise Problem 2

Assuming that Young's modulus varies with z, (i.e., $E = E(z)$), derive the governing equations of the Timoshenko beam in terms of the generalized displacements (u, w, ϕ_x).

$$N_{xx} = \int_A \sigma_{xx}\, dA = A_{xx}\frac{\partial u}{\partial x} + B_{xx}\frac{\partial \phi_x}{\partial x}$$

$$M_{xx} = \int_A z\sigma_{xx}\, dA = B_{xx}\frac{\partial u}{\partial x} + D_{xx}\frac{\partial \phi_x}{\partial x}, \quad S_{xx} = K_s\int_A G(z)\, dA$$

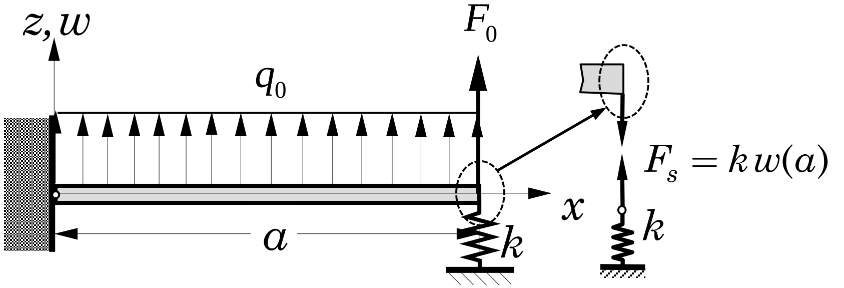
$$\frac{\partial}{\partial x}\left(A_{xx}\frac{\partial u}{\partial x} + B_{xx}\frac{\partial \phi_x}{\partial x}\right) + f = 0$$

$$\frac{\partial}{\partial x}\left[K_s S_{xz}\left(\phi_x + \frac{\partial w}{\partial x}\right)\right] + q = 0$$

$$\frac{\partial}{\partial x}\left(B_{xx}\frac{\partial u}{\partial x} + D_{xx}\frac{\partial \phi_x}{\partial x}\right) - \left[K_s S_{xz}\left(\phi_x + \frac{\partial w}{\partial x}\right)\right] = 0$$

Exercise Problem 3:

Consider a beam (with constant *EI*) fixed at the left end, spring-supported at the right end, and subjected to uniform load of intensity q_0 distributed over the entire span and a point load F_0 at the right end, as shown in Fig. Assuming that the spring is linearly elastic with a Spring constant *k*, determine the elongation of the spring.



$$w(x)=\frac{q_0x^4}{24EI}+C_1\frac{x^3}{6EI}+C_2\frac{x^2}{2EI}+C_3x+C_4$$

at $x=0: w=0,\ \frac{dw}{dx}=0;$

at $x=a: M=0,\ V=F_0-F_s$

$$C_3=C_4=0, C_1=-q_0a-F_0+F_s$$

$F_s=kw(a)$

$$C_2=\frac{q_0a^2}{2}+F_0a-F_sa$$

$$w(x)=\frac{q_0x^4}{24EI}-\left(q_0a+F_0-F_s\right)\frac{x^3}{6EI}+\left(\frac{q_0a^2}{2}+F_0a-F_sa\right)\frac{x^2}{2EI}$$

$$\delta=w(a)=\left(\frac{q_0a^4}{8EI}+\frac{F_0a^3}{3EI}\right)\left(1+\frac{ka^3}{3EI}\right)^{-1}$$

Exercise Problem 4

Determine the Navier solution for the case in which Young's modulus varies with z, (i.e., $E = E(z)$). Use the Euler-Bernoulli beam theory.

$$\frac{\partial}{\partial x}\left(A_{xx}\frac{\partial u}{\partial x}+B_{xx}\frac{\partial \theta_x}{\partial x}\right)+f=0;\ \frac{\partial^2}{\partial x^2}\left(B_{xx}\frac{\partial u}{\partial x}+D_{xx}\frac{\partial \theta_x}{\partial x}\right)+q=0$$

$$u=\sum_{n=1}^{\infty}U_n\cos\alpha_n x,\ w=\sum_{n=1}^{\infty}W_n\sin\alpha_n x,\quad \alpha_n=\frac{n\pi}{a}$$

$$\sum_{n=1}^{\infty}\left(-\alpha_n^2 A_{xx}U_n+\alpha_n^3 B_{xx}W_n+F_n\right)\cos\alpha_n x=0$$

$$\sum_{n=1}^{\infty}\left(\alpha_n^3 B_{xx}U_n+\alpha_n^4 D_{xx}+Q_n\right)\sin\alpha_n x=0$$

$$\begin{bmatrix}-\alpha_n^2 A_{xx} & \alpha_n^3 B_{xx}\\ \alpha_n^3 B_{xx} & \alpha_n^4 D_{xx}\end{bmatrix}\begin{Bmatrix}U_n\\ W_n\end{Bmatrix}=-\begin{Bmatrix}F_n\\ Q_n\end{Bmatrix}$$

Exercise Problem 5

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Determine the Navier solution for the case in which Young's modulus varies with z, (i.e., $E = E(z)$). Use the Timoshenko beam theory.

$$\frac{\partial}{\partial x}\left(A_{xx}\frac{\partial u}{\partial x} + B_{xx}\frac{\partial \phi_x}{\partial x}\right) + f = 0, \quad \frac{\partial}{\partial x}\left[K_s S_{xz}\left(\phi_x + \frac{\partial w}{\partial x}\right)\right] + q = 0$$

$$\frac{\partial}{\partial x}\left(B_{xx}\frac{\partial u}{\partial x} + D_{xx}\frac{\partial \phi_x}{\partial x}\right) - \left[K_s S_{xz}\left(\phi_x + \frac{\partial w}{\partial x}\right)\right] = 0$$

$$u = \sum_{n=1}^{\infty} U_n \cos\alpha_n x, \; w = \sum_{n=1}^{\infty} W_n \sin\alpha_n x, \;\; \phi = \sum_{n=1}^{\infty} X_n \cos\alpha_n x$$

$$\begin{bmatrix} -\alpha_n^2 A_{xx} & 0 & -\alpha_n^2 B_{xx} \\ 0 & K_s S_{xz}\alpha_n^2 & -K_s S_{xz}\alpha_n \\ -\alpha_n^2 B_{xx} & -K_s S_{xz}\alpha_n & -\left(\alpha_n^2 D_{xx} + K_s S_{xz}\right) \end{bmatrix} \begin{Bmatrix} U_n \\ W_n \\ X_n \end{Bmatrix} = -\begin{Bmatrix} F_n \\ -Q_n \\ 0 \end{Bmatrix}$$