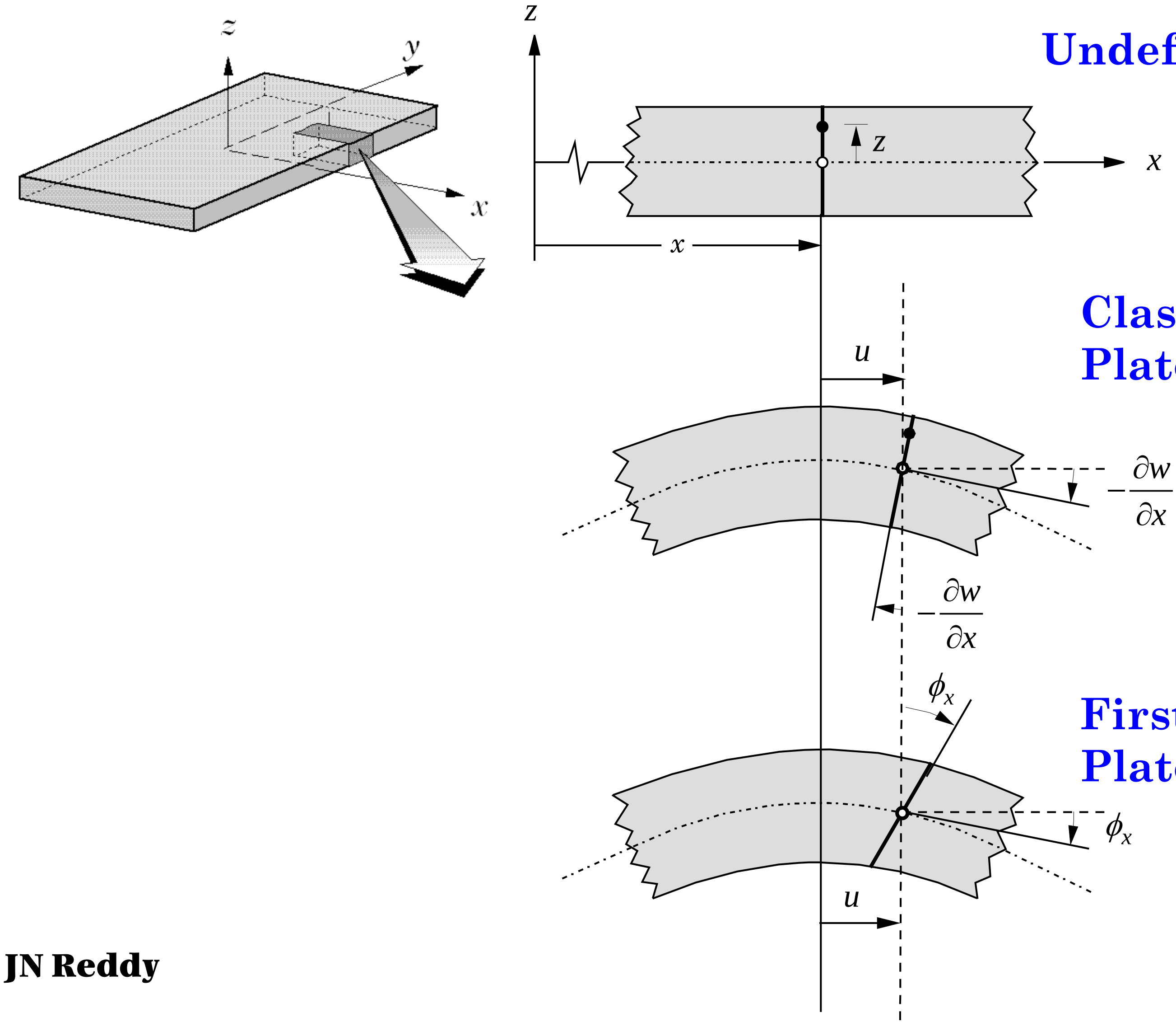
LINEAR ANALYSIS OF FSDT PLATES

CONTENTS

- Displacement field
- Strains
- Equations of motion using vector approach
- Equilibrium equations using the principle of virtual displacements
- Boundary conditions
- Stress resultants in terms of the generalized displacements

Kinematics of the Classical and Shear Deformation Plate Theories



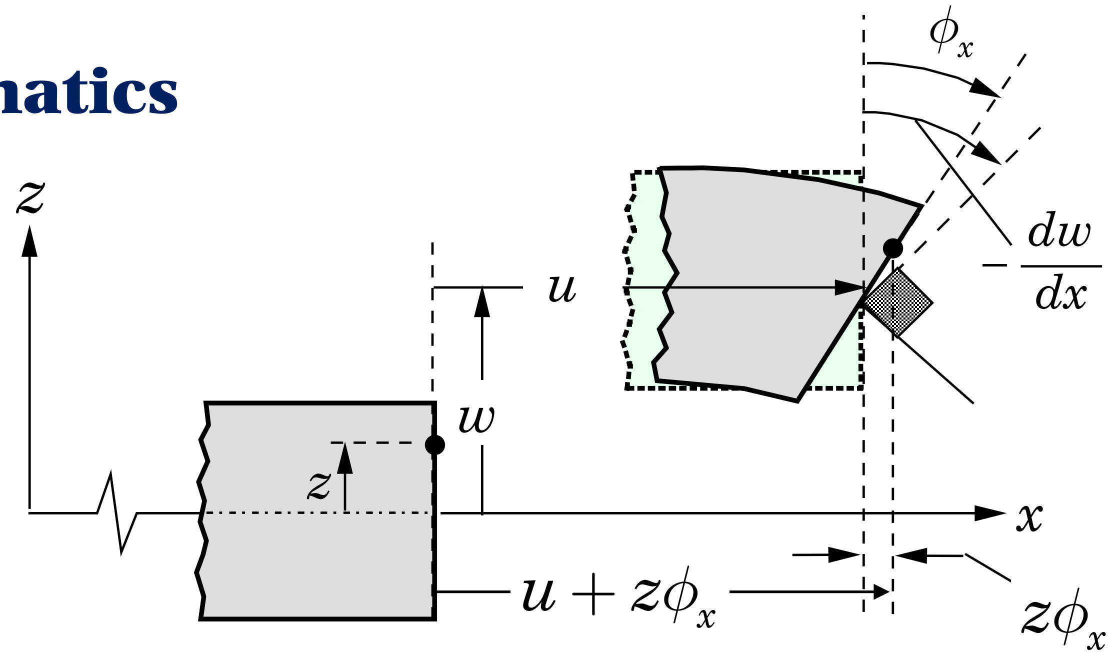
Undeformed Edge

Classical (Kirchhoff) Plate Theory (CPT)

First-Order (Mindlin) Plate Theory (FST)

KINEMATICS AND DISPLACEMENTS

Kinematics



Displacement Field

$$u_1(x,y,z,t) = u(x,y,t) + z\phi_x(x,y,t)$$
$$u_2(x,y,z,t) = v(x,y,t) + z\phi_y(x,y,t)$$
$$u_3(x,y,z,t) = w(x,y,t)$$

LINEAR SYRAINS

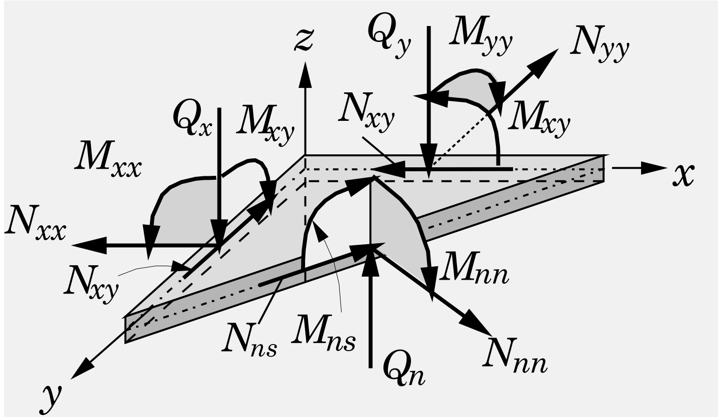
Linear strains

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ 2\varepsilon_{xy} \\ 2\varepsilon_{yz} \\ 2\varepsilon_{xz} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} + z\dfrac{\partial \phi_x}{\partial x} \\ \dfrac{\partial v}{\partial y} + z\dfrac{\partial \phi_y}{\partial y} \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} + z\left(\dfrac{\partial \phi_x}{\partial y} + \dfrac{\partial \phi_y}{\partial x}\right) \\ \phi_x + \dfrac{\partial w}{\partial x} \\ \phi_y + \dfrac{\partial w}{\partial y} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{xx}^0 + z\varepsilon_{xx}^1 \\ \varepsilon_{yy}^0 + z\varepsilon_{yy}^1 \\ 2\varepsilon_{xy}^0 + 2z\varepsilon_{xy}^1 \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}$$

STRESS RESULTANTS

Stress resultants



$$N_{xx} = \int_{-h/2}^{h/2} \sigma_{xx}\, dz, \quad N_{yy} = \int_{-h/2}^{h/2} \sigma_{yy}\, dz,$$

$$N_{xy} = \int_{-h/2}^{h/2} \sigma_{xy}\, dz, \quad Q_x = \int_{-h/2}^{h/2} \sigma_{xz}\, dz,$$

$$Q_y = \int_{-h/2}^{h/2} \sigma_{yz}\, dz, \quad M_{xx} = \int_{-h/2}^{h/2} z\sigma_{xx}\, dz,$$

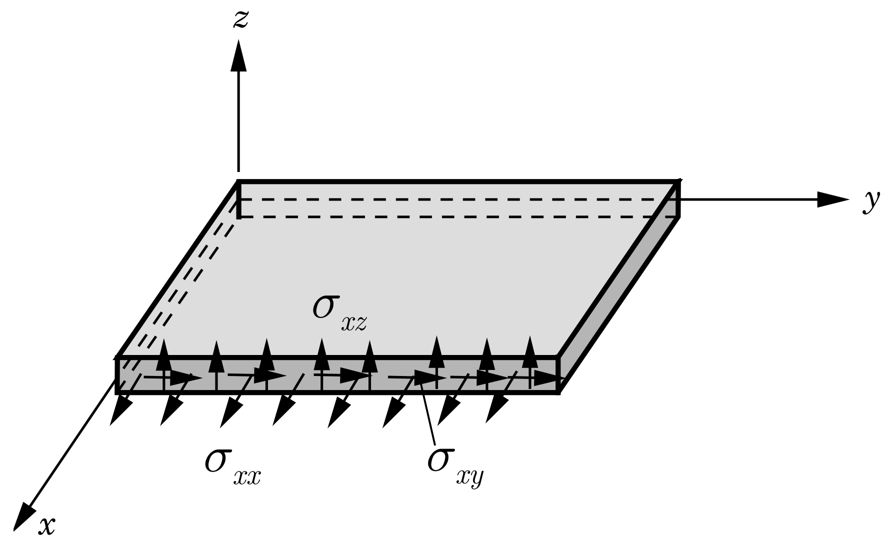
$$M_{yy} = \int_{-h/2}^{h/2} z\sigma_{yy}\, dz, \quad M_{xy} = \int_{-h/2}^{h/2} z\sigma_{xy}\, dz, \; N$$

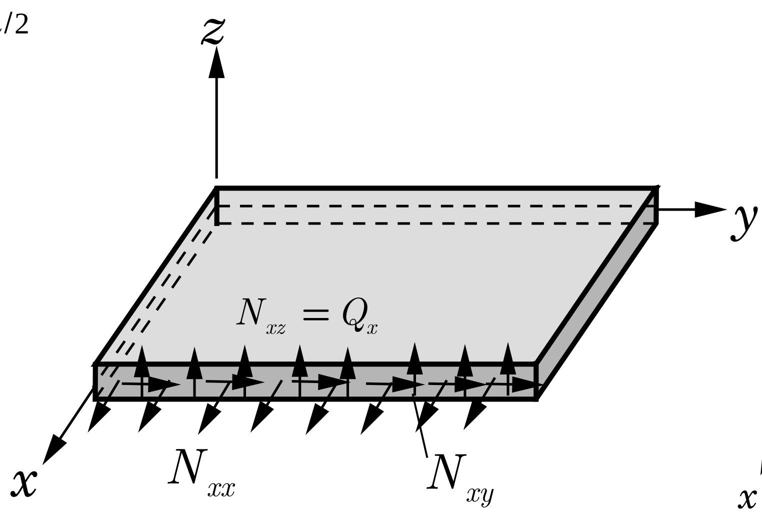
STRESSES AND STRESS RESULTANTS ON AN EDGE OF A PLATE

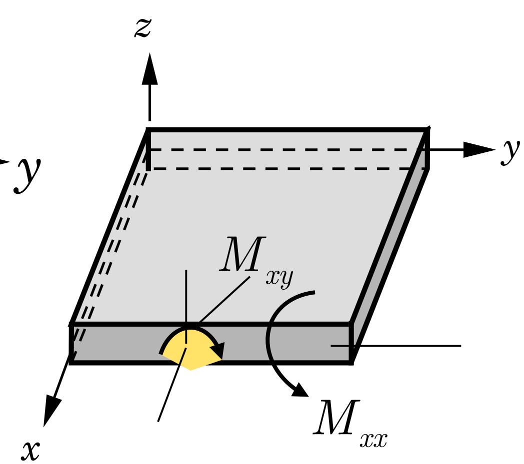
$$N_{xx} = \int_{-h/2}^{h/2} \sigma_{xx}\, dz, \quad N_{xy} = \int_{-h/2}^{h/2} \sigma_{xy}\, dz,$$

$$Q_x = \int_{-h/2}^{h/2} \sigma_{xz}\, dz, \quad M_{xx} = \int_{-h/2}^{h/2} z\sigma_{xx}\, dz,$$

$$M_{xy} = \int_{-h/2}^{h/2} z\sigma_{xy}\, dz$$



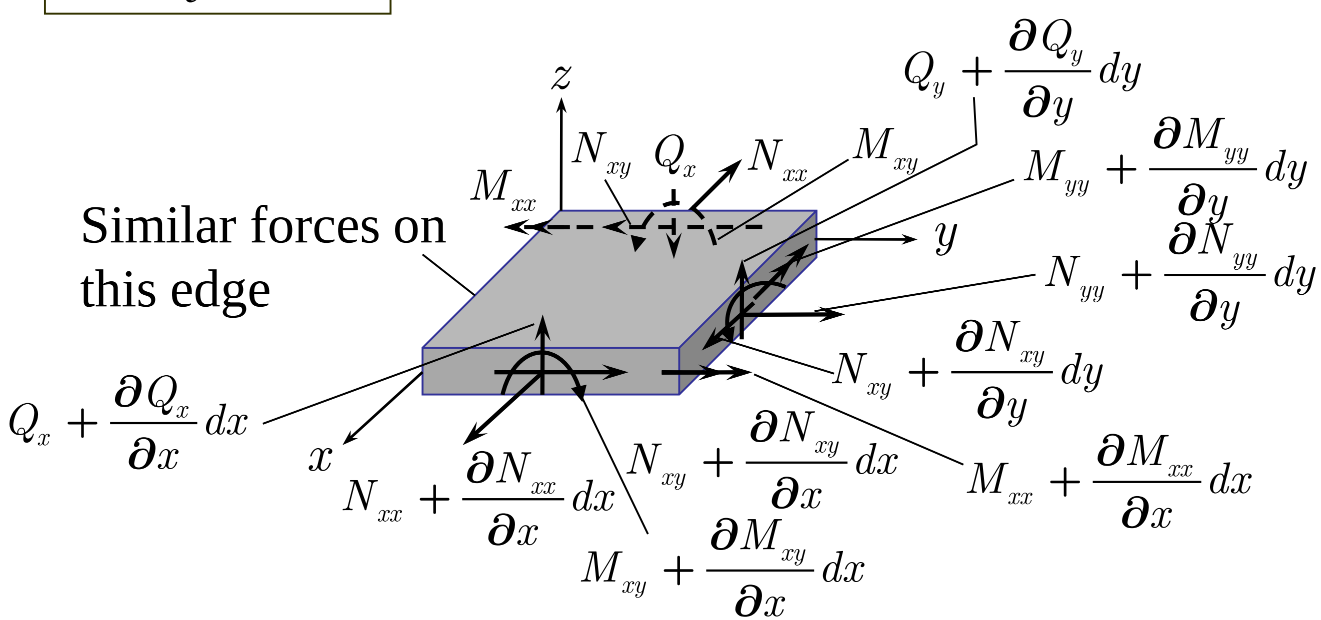




EQUATIONS OF EQUILIBRIUM

(vector approach)

Element of dimensions dx, dy, and h



Equilibrium of a plate element

EQUATIONS OF EQUILIBRIUM (vector approach)

Equations of motion (FSDT)

$$\sum F_x = 0: \quad \frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} + f_x = I_0 \frac{\partial^2 u}{\partial t^2}$$

$$\sum F_y = 0: \quad \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} + f_y = I_0 \frac{\partial^2 v}{\partial t^2}$$

$$\sum F_z = 0: \quad \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q = I_0 \frac{\partial^2 w}{\partial t^2}$$

$$\sum M_y = 0: \quad \frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = I_2 \frac{\partial^2 \phi_x}{\partial t^2}$$

$$\sum M_x = 0: \quad \frac{\partial M_{xy}}{\partial x} + \frac{\partial M_{yy}}{\partial y} - Q_y = I_2 \frac{\partial^2 \phi_y}{\partial t^2}$$

$$I_0 = \int_{-h/2}^{h/2} \rho \, dz = \rho h, \qquad I_2 = \int_{-h/2}^{h/2} \rho \, z^2 dz = \frac{\rho h^3}{12}$$

PRINCIPLE OF VIRTUAL DISPLACEMENTS

$$0 = \delta W^e \equiv \delta W_I^e + \delta W_E^e$$

$$\delta W_I^e = \int_V \sigma_{ij}\,\delta\varepsilon_{ij}\,dV$$

$$= \int_V \left(\sigma_{11}\,\delta\varepsilon_{11} + \sigma_{22}\,\delta\varepsilon_{22} + \sigma_{12}\,2\delta\varepsilon_{12} + \sigma_{13}\,2\delta\varepsilon_{13} + \sigma_{23}\,2\delta\varepsilon_{23}\right) dV$$

$$= \int_\Omega \left\{ \int_{-\frac{h}{2}}^{\frac{h}{2}} \left[\sigma_{xx}\left(\delta\varepsilon_{xx}^0 + z\delta\varepsilon_{xx}^1\right) + \sigma_{yy}\left(\delta\varepsilon_{yy}^0 + z\delta\varepsilon_{yy}^1\right) \right.\right.$$

$$+ \sigma_{xy}\left(\delta\gamma_{xy}^0 + z\delta\gamma_{xy}^1\right) + K_s\sigma_{xz}\delta\gamma_{xz}^0$$

$$\left.\left. + K_s\sigma_{yz}\delta\gamma_{yz}^0 \right] dz \right\} dx\,dy$$

$$\delta W_E^e = -\left\{ \oint_\Gamma \int_{-\frac{h}{2}}^{\frac{h}{2}} \left[\sigma_{nn}\left(\delta u_n + z\delta\phi_n\right) + \sigma_{ns}\left(\delta u_s + z\delta\phi_s\right) + \sigma_{nz}\delta w \right] dz\,ds \right.$$

$$\left. + \int_\Omega q\,\delta w\,dx\,dy \right\}$$

PRINCIPLE OF VIRTUAL DISPLACEMENTS

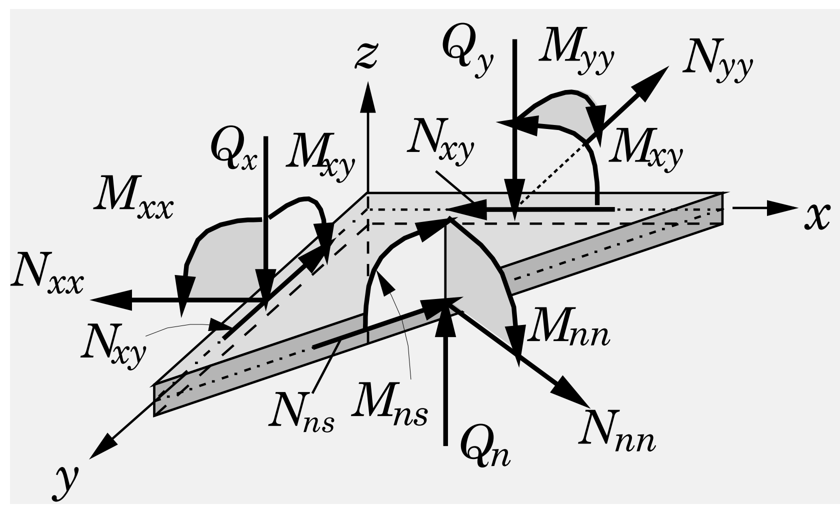
$$0 = \int_{\Omega}\Big[N_{xx}\delta\varepsilon_{xx}^{0} + M_{xx}\delta\varepsilon_{xx}^{1} + N_{yy}\delta\varepsilon_{yy}^{0} + M_{yy}\delta\varepsilon_{yy}^{1} + N_{xy}\delta\gamma_{xy}^{0} + M_{xy}\delta\gamma_{xy}^{1}$$

$$+Q_{x}\delta\gamma_{xz}^{0} + Q_{y}\delta\gamma_{yz}^{0} - q\delta w\Big]dx\,dy$$

$$-\oint_{\Gamma}\left(N_{nn}\delta u_{n} + N_{ns}\delta u_{s} + M_{nn}\delta\phi_{n} + M_{ns}\delta\phi_{s} + Q_{n}\delta w\right)ds$$

$$\left\{\begin{array}{c} N_{xx} \\ N_{yy} \\ N_{xy} \end{array}\right\} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \left\{\begin{array}{c} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{array}\right\}$$

$$\left\{\begin{array}{c} M_{xx} \\ M_{yy} \\ M_{xy} \end{array}\right\} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \left\{\begin{array}{c} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{array}\right\} z\ dz$$



$$(Q_{x},Q_{y}) = \int_{-h/2}^{h/2}(\sigma_{xz},\sigma_{xz})dz$$

PRINCIPLE OF VIRTUAL DISPLACEMENTS

$$0=\int_{\Omega}\left[N_{xx}\frac{\partial\delta u}{\partial x}+M_{xx}\frac{\partial\delta\phi_x}{\partial x}+N_{yy}\frac{\partial\delta v}{\partial y}+M_{yy}\frac{\partial\delta\phi_y}{\partial y}+N_{xy}\left(\frac{\partial\delta u}{\partial y}+\frac{\partial\delta v}{\partial x}\right)\right.$$

$$\left.+M_{xy}\left(\frac{\partial\delta\phi_x}{\partial y}+\frac{\partial\delta\phi_y}{\partial x}\right)+Q_x\left(\delta\phi_x+\frac{\partial\delta w}{\partial x}\right)+Q_y\left(\delta\phi_y+\frac{\partial\delta w}{\partial y}\right)\right]dx\,dy$$

$$-\int_{\Omega}(f_x\delta u+f_y\delta v+q\delta w)\;dxdy-\oint_{\Gamma}\left(N_{nn}\delta u_n+N_{ns}\delta u_s+M_{nn}\delta\phi_n+M_{ns}\delta\phi_s+Q_n\delta w\right)ds$$

$$\frac{\partial N_{xx}}{\partial x}+\frac{\partial N_{xy}}{\partial y}+f_x=0,\quad \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_{yy}}{\partial y}+f_y=0,$$

$$\frac{\partial M_{xx}}{\partial x}+\frac{\partial M_{xy}}{\partial y}-Q_x=0,\quad \frac{\partial M_{xy}}{\partial x}+\frac{\partial M_{yy}}{\partial y}-Q_y=0,$$

$$\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}+q=0$$

CONSTITUTIVE RELATIONS

Stress-Strain Relations

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}, \quad \begin{Bmatrix} \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} = \begin{bmatrix} Q_{55} & 0 \\ 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}$$

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}, Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}}, Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}},$$

$$Q_{66} = G_{12}, Q_{44} = G_{23}, Q_{55} = G_{13},$$

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ \dfrac{\partial v}{\partial y} \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \end{Bmatrix} + z \begin{Bmatrix} \dfrac{\partial \phi_x}{\partial x} \\ \dfrac{\partial \phi_y}{\partial y} \\ \dfrac{\partial \phi_x}{\partial y} + \dfrac{\partial \phi_y}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \phi_x + \dfrac{\partial w}{\partial x} \\ \phi_y + \dfrac{\partial w}{\partial y} \end{Bmatrix}$$

Stress Resultant-Displacement Relations

$$N_{xx} = \int_{-h/2}^{h/2} \sigma_{xx}\, dz = \int_{-h/2}^{h/2} \left\{ Q_{11}\left(\frac{\partial u}{\partial x} + z\frac{\partial \phi_x}{\partial x}\right) + Q_{12}\left(\frac{\partial v}{\partial y} + z\frac{\partial \phi_y}{\partial y}\right)\right\} dz$$

$$= A_{11}\frac{\partial u}{\partial x} + A_{12}\frac{\partial v}{\partial y}$$

$$N_{xy} = \int_{-h/2}^{h/2} \sigma_{xy}\, dz = \int_{-h/2}^{h/2} Q_{66}\left[\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + z\left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x}\right)\right] dz$$

$$= Q_{66}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$M_{xx} = \int_{-h/2}^{h/2} \sigma_{xx}\, z\; dz = \int_{-h/2}^{h/2} z\left[Q_{11}\left(\frac{\partial u}{\partial x} + z\frac{\partial \phi_x}{\partial x}\right) + Q_{12}\left(\frac{\partial v}{\partial y} + \frac{\partial v}{\partial y} + z\frac{\partial \phi_y}{\partial y}\right)\right] dz$$

$$= D_{11}\frac{\partial \phi_x}{\partial x} + D_{12}\frac{\partial \phi_y}{\partial y}$$

$$M_{xy} = \int_{-h/2}^{h/2} \sigma_{xy}\, z\; dz = \int_{-h/2}^{h/2} z\; Q_{66}\left[\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + z\left(+\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x}\right)\right] dz$$

$$= D_{66}\left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x}\right)$$

JN Reddy

STRESS RESULTANTS IN TERMS OF THE GENERALIZED DISPLACEMENTS

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon^0_{xx} \\ \varepsilon^0_{yy} \\ 2\varepsilon^0_{xy} \end{Bmatrix}, \quad \begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon^1_{xx} \\ \varepsilon^1_{yy} \\ 2\varepsilon^1_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \begin{bmatrix} A_{55} & 0 \\ 0 & A_{44} \end{bmatrix} \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}, \quad (A_{ij}, D_{ij}) = \int_{-h/2}^{h/2} Q_{ij}(1, z^2)\, dz$$

Plate stiffnesses

$$A_{11} = \frac{E_1 h}{1 - \nu_{12}\nu_{21}}, \quad A_{12} = \frac{\nu_{12} E_2 h}{1 - \nu_{12}\nu_{21}}, \quad A_{22} = \frac{E_2 h}{1 - \nu_{12}\nu_{21}},$$

$$D_{11} = \frac{E_1 h^3}{12(1 - \nu_{12}\nu_{21})}, \quad D_{12} = \frac{\nu_{12} E_2 h^3}{12(1 - \nu_{12}\nu_{21})}, \quad D_{22} = \frac{E_2 h^3}{12(1 - \nu_{12}\nu_{21})},$$

$$D_{66} = \frac{G_{12} h^3}{12}, A_{44} = G_{23} h, \quad A_{55} = G_{13} h, \quad A_{66} = G_{12} h$$

Equilibrium Equations in terms of the Generalized Displacements

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{0} \\ \varepsilon_{yy}^{0} \\ 2\varepsilon_{xy}^{0} \end{Bmatrix}, \quad \begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{1} \\ \varepsilon_{yy}^{1} \\ 2\varepsilon_{xy}^{1} \end{Bmatrix}$$

$$\begin{Bmatrix} Q_{x} \\ Q_{y} \end{Bmatrix} = \begin{bmatrix} A_{55} & 0 \\ 0 & A_{44} \end{bmatrix} \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}, \quad (A_{ij}, D_{ij}) = \int_{-h/2}^{h/2} Q_{ij}(1, z)\, d$$

$$\frac{\partial}{\partial x}\left(A_{11}\frac{\partial u}{\partial x} + A_{12}\frac{\partial v}{\partial y}\right) + \frac{\partial}{\partial y}\left(A_{66}\frac{\partial u}{\partial y} + A_{66}\frac{\partial v}{\partial x}\right) + f_x = 0$$

$$\frac{\partial}{\partial x}\left(A_{66}\frac{\partial u}{\partial y} + A_{66}\frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(A_{12}\frac{\partial u}{\partial x} + A_{22}\frac{\partial v}{\partial y}\right) + f_y = 0$$

$$\frac{\partial}{\partial x}\left[A_{55}\left(\phi_x + \frac{\partial w}{\partial x}\right)\right] + \frac{\partial}{\partial y}\left[A_{44}\left(\phi_y + \frac{\partial w}{\partial y}\right)\right] + q = 0$$

$$\frac{\partial}{\partial x}\left(D_{11}\frac{\partial \phi_x}{\partial x} + D_{12}\frac{\partial \phi_y}{\partial y}\right) + \frac{\partial}{\partial y}\left(D_{66}\frac{\partial \phi_x}{\partial y} + D_{66}\frac{\partial \phi_y}{\partial x}\right) - A_{55}\left(\phi_x + \frac{\partial w}{\partial x}\right) = 0$$

$$\frac{\partial}{\partial x}\left(D_{66}\frac{\partial \phi_x}{\partial y} + D_{66}\frac{\partial \phi_y}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_{12}\frac{\partial \phi_x}{\partial x} + D_{22}\frac{\partial \phi_y}{\partial y}\right) - A_{44}\left(\phi_y + \frac{\partial w}{\partial y}\right) = 0$$

Navier Solution of Simply Supported ORTHOTROPIC PLATES

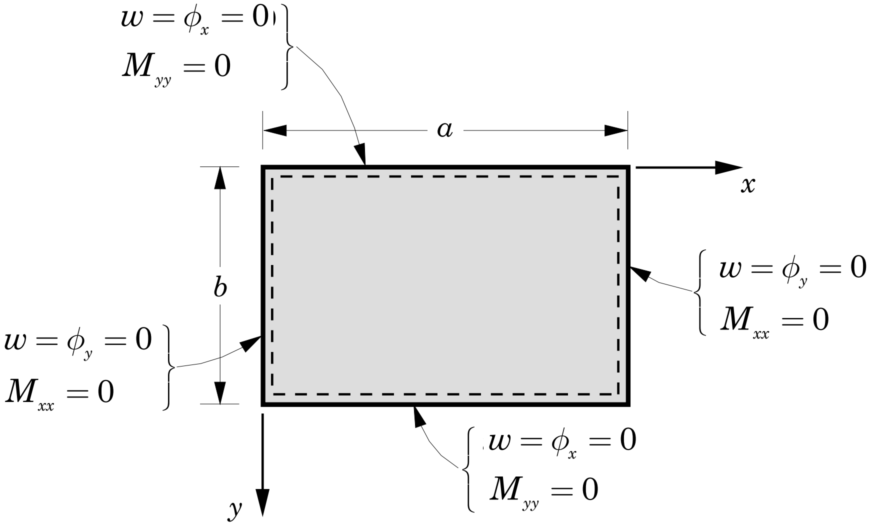
$$w(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{mn}\sin\alpha_m x\sin\beta_n y$$

$$\phi_x(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}X_{mn}\cos\alpha_m x\sin\beta_n y$$

$$\phi_y(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}Y_{mn}\sin\alpha_m x\cos\beta_n y$$

$$q(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}Q_{mn}\sin\alpha_m x\sin\beta_n y$$

$$Q_{mn}=\frac{4}{ab}\int_0^a\int_0^b q(x,y)\sin\alpha_m x\sin\beta_n y\,dxdy$$



Substitution of the expansions into the equations of equilibrium give the following algebraic equations for the coefficients of the expansion:

$$\begin{bmatrix} S_{11} & S_{12} & S_{13}\\ S_{12} & S_{22} & S_{23}\\ S_{13} & S_{23} & S_{33}\end{bmatrix}\begin{Bmatrix} W_{mn}\\ X_{mn}\\ Y_{mn}\end{Bmatrix}=\begin{Bmatrix} Q_{mn}\\ 0\\ 0\end{Bmatrix}$$

Navier Solution of Simply Supported Orthotropic Plates (continued)

$$s_{11} = K_s(A_{55}\alpha_m^2 + A_{44}\beta_n^2) + k, s_{12} = K_s A_{55}\alpha_m, \; s_{13} = K_s A_{44}\beta_n,$$

$$s_{22} = (D_{11}\alpha_m^2 + D_{66}\beta_n^2 + K_s A_{55}), s_{23} = (D_{12} + D_{66})\alpha_m\beta_n,$$

$$s_{33} = (D_{66}\alpha_m^2 + D_{22}\beta_n^2 + K_s A_{44})$$

The solution becomes

$$W_{mn} = \frac{b_0 Q_{mn}}{b_{mn}}, \; X_{mn} = \frac{b_1 W_{mn}}{b_0}, \; Y_{mn} = \frac{b_2 W_{mn}}{b_0}$$

$$b_{mn} = s_{11}b_0 + s_{12}b_1 + s_{13}b_2 \;, \quad b_0 = s_{22}s_{33} - s_{23}s_{23},$$

$$b_1 = s_{23}s_{13} - s_{12}s_{33}, \qquad b_2 = s_{12}s_{23} - s_{22}s_{13},$$

$$M_{xx} = -\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(D_{11}\alpha_m X_{mn} + D_{12}\beta_n Y_{mn}\right)\sin\alpha_m x \, \sin\beta_n y,$$

$$M_{yy} = -\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(D_{12}\alpha_m X_{mn} + D_{22}\beta Y_{mn}\right)\sin\alpha_m x \sin\beta_n y,$$

$$M_{xy} = D_{66}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(\beta_n X_{mn} + \alpha_m Y_{mn}\right)\cos\alpha_m x \, \cos\beta_n y$$

Navier Solution of Simply Supported Orthotropic Plates

$$E_1 = 25E_2,\ G_{12} = G_{13} = 0.5E_2, G_{23} = 0.2E_2, \nu_{12} = 0.25, K_s = 5/6$$

Load	$\frac{b}{h}$	$\bar{w}$	$\bar{\sigma}_{xx}$	$\bar{\sigma}_{yy}$	$\bar{\sigma}_{xy}$	$\bar{\sigma}_{xz}$	$\bar{\sigma}_{yz}$
Isotropic plates ($\nu = 0.25$)							
SL	10	0.2702	0.1900	0.1900	0.1140	0.1910	0.1910
						0.2387	0.2387†
	20	0.2600	0.1900	0.1900	0.1140	0.1910	0.1910
	50	0.2572	0.1900	0.1900	0.1140	0.1910	0.1910
	100	0.2568	0.1900	0.1900	0.1140	0.1910	0.1910
	CPT	0.2566	0.1900	0.1900	0.1140	–	–
UL (19)	10	0.4259	0.2762	0.2762	0.2085	0.3927	0.3927
						0.4909	0.4909†
	20	0.4111	0.2762	0.2762	0.2085	0.3927	0.3927
	50	0.4070	0.2762	0.2762	0.2085	0.3927	0.3927
	100	0.4060	0.2762	0.2762	0.2085	0.3927	0.3927
	CPT	0.4062	0.2762	0.2762	0.2085	–	–

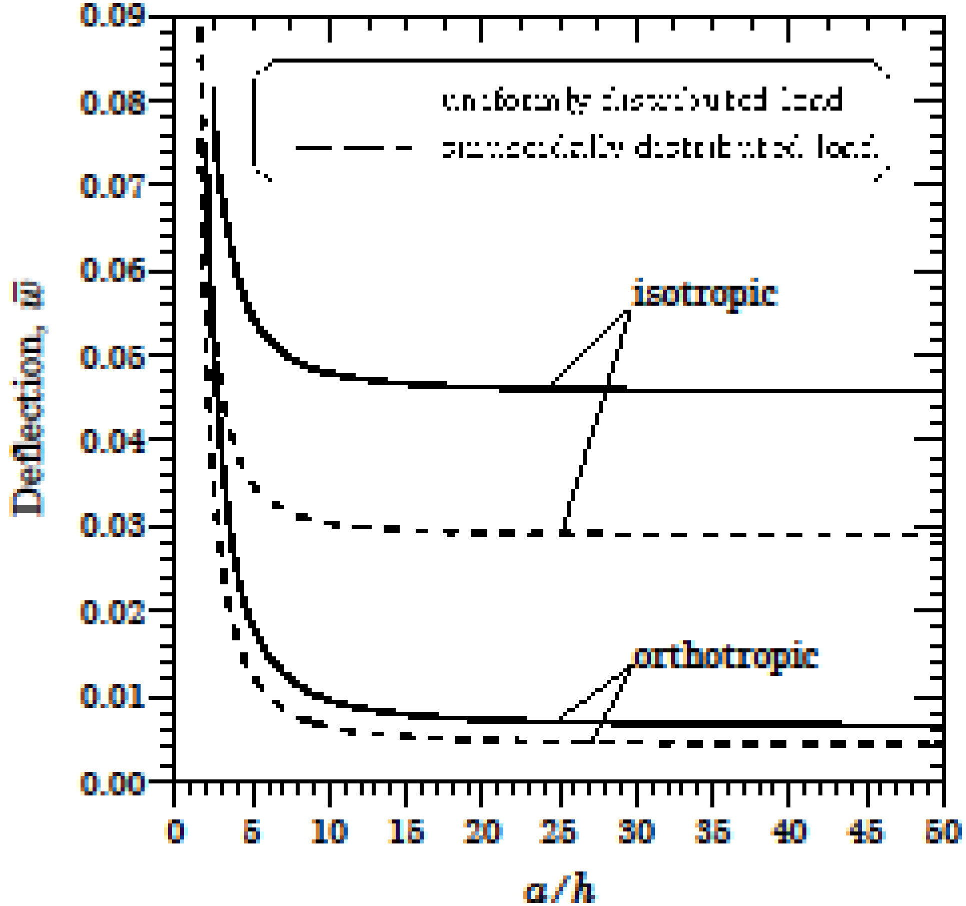
$$\bar{w} = w_0\left(E_2 h^3 / b^4 q_0\right), \hat{w} = w_0\left(D_{22} / b^4 q_0\right)\times 10^2,\ \bar{\sigma}_{xx} = \sigma_{xx}\left(h^2 / b^2 q_0\right),\ \bar{\sigma}_{yy} = \sigma_{yy}\left(h^2 / b^2 q_0\right),$$

$$\bar{\sigma}_{xy} = \sigma_{xy}\left(h^2 / b^2 q_0\right),\ \bar{\sigma}_{xz} = \sigma_{xz}\left(h / b q_0\right),\ \bar{\sigma}_{yz} = \sigma_{yz}\left(h / b q_0\right)$$

$$\bar{\sigma}_{xx}(a/2, b/2, \frac{h}{2}),\ \bar{\sigma}_{yy}(a/2, b/2, \frac{h}{2}),\ \bar{\sigma}_{xy}(a, b, -\frac{h}{2}), \bar{\sigma}_{xz}(0, b/2, \frac{h}{2}),\ \bar{\sigma}_{yz}(a/2, 0, \frac{h}{2})$$

Navier Solution of Simply Supported Orthotropic Plates

Load	$\frac{a}{h}$	$\bar{w}$	$\bar{\sigma}_{xx}$	$\bar{\sigma}_{yy}$	$\bar{\sigma}_{xy}$	$\bar{\sigma}_{xz}$	$\bar{\sigma}_{yz}$
Orthotropic plates							
SL	10	0.0533	0.5248	0.0338	0.0246	0.3452	0.0367
						0.4315	0.0459
	20	0.0404	0.5350	0.0286	0.0222	0.3501	0.0319
						0.4376	0.0399
	50	0.0367	0.5380	0.0270	0.0214	0.3515	0.0304
						0.4394	0.0380
	100	0.0362	0.5385	0.0267	0.0213	0.3517	0.0302
						0.4397	0.0377
	CPT	0.0360	0.5387	0.0267	0.0213	—	—
						0.4398	0.0376
UL (19)	10	0.0795	0.7706	0.0352	0.0539	0.6147	0.1529
						0.7684	0.1911
	20	0.0607	0.7828	0.0272	0.0487	0.6194	0.1466
						0.7742	0.1833
	50	0.0553	0.7860	0.0249	0.0468	0.6207	0.1452
						0.7756	0.1814
	100	0.0545	0.7865	0.0245	0.0464	0.6206	0.1449
						0.7757	0.1812
	CPT	0.0543	0.7866	0.0244	0.0463	—	—
						0.7758	0.1811



Nondimensional center transverse deflection ($\overline{w}$) versus side-to-thickness ratio (*a/h*) for simply supported square plates.