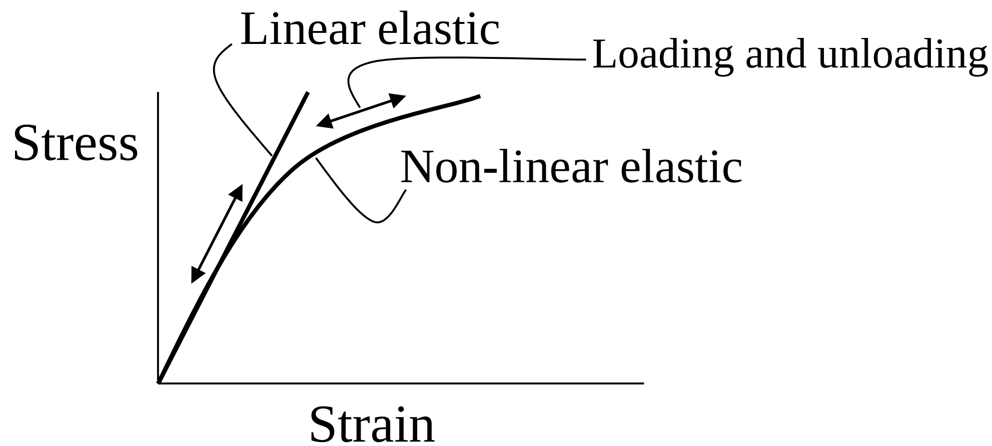
Nonlinear Elasticity and Plasticity

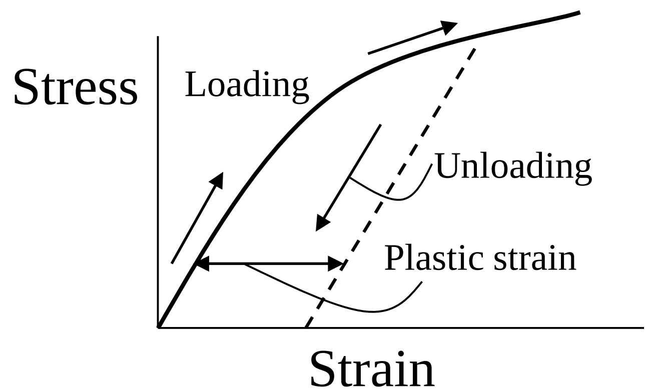
- Nonlinear elasticity
- Plasticity
- Ideal plasticity and strain hardening plasticity
- Stress-strain curves
- Finite element models of nonlinear elasticity
- Numerical examples
- Small deformation theory of plasticity
- Finite element formulation
- Numerical examples

Nonlinear Elasticity and Plasticity

Nonlinear Elasticity

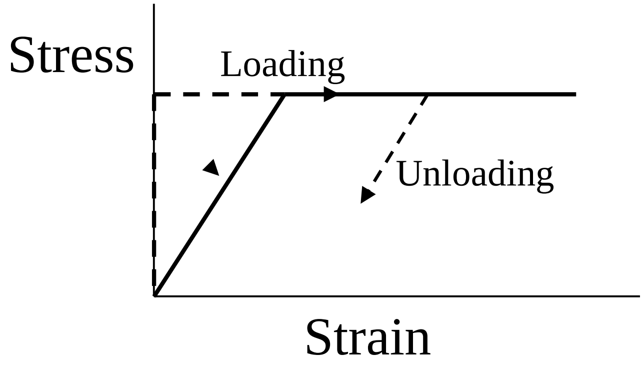


Plasticity

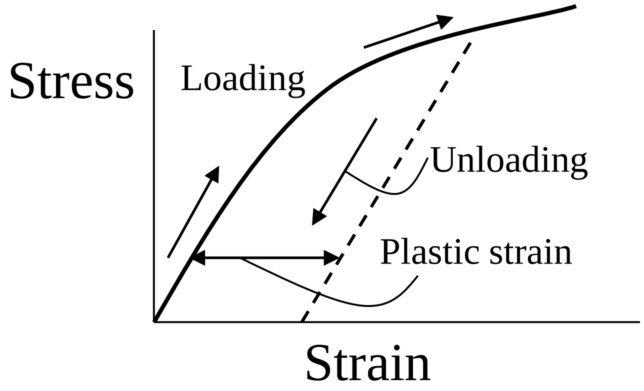


Ideal Plasticity and Strain Hardening Plasticity

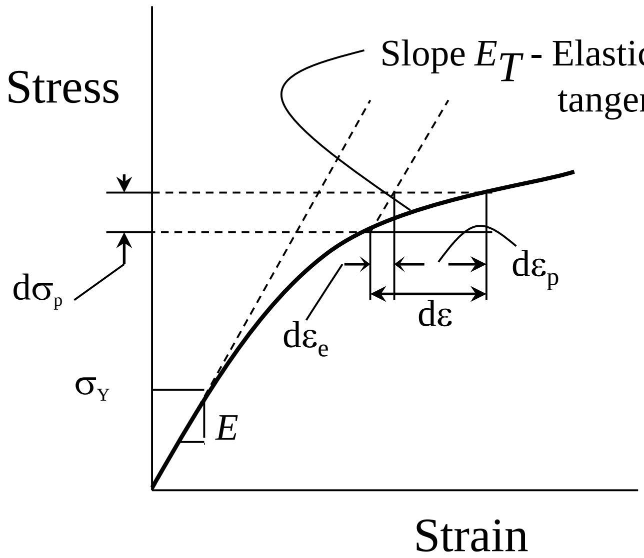
Ideal (or Perfect) Plasticity

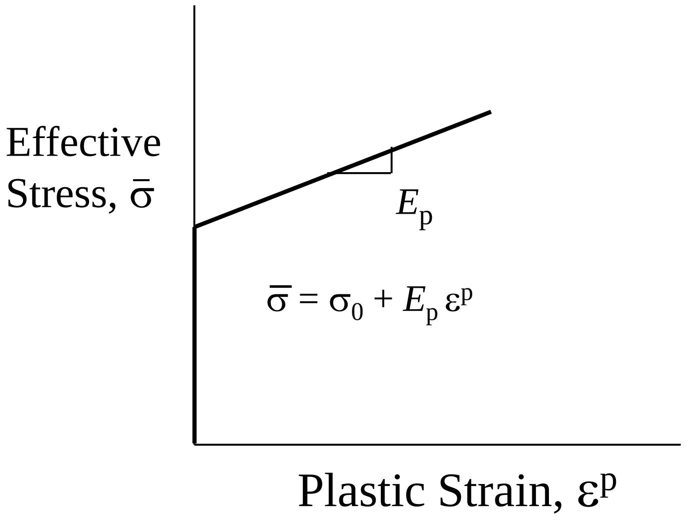


Strain Hardening Plasticity

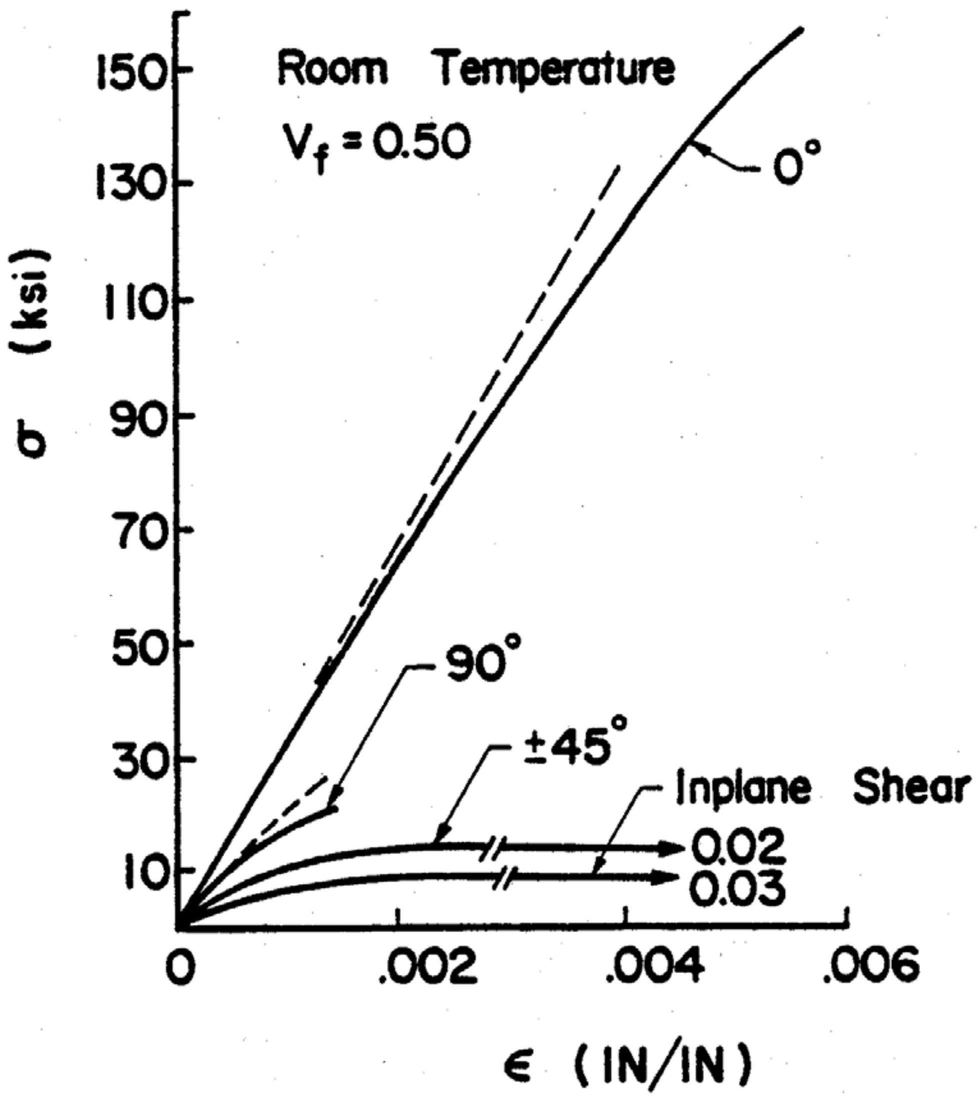


Ideal Plasticity and Strain Hardening Plasticity

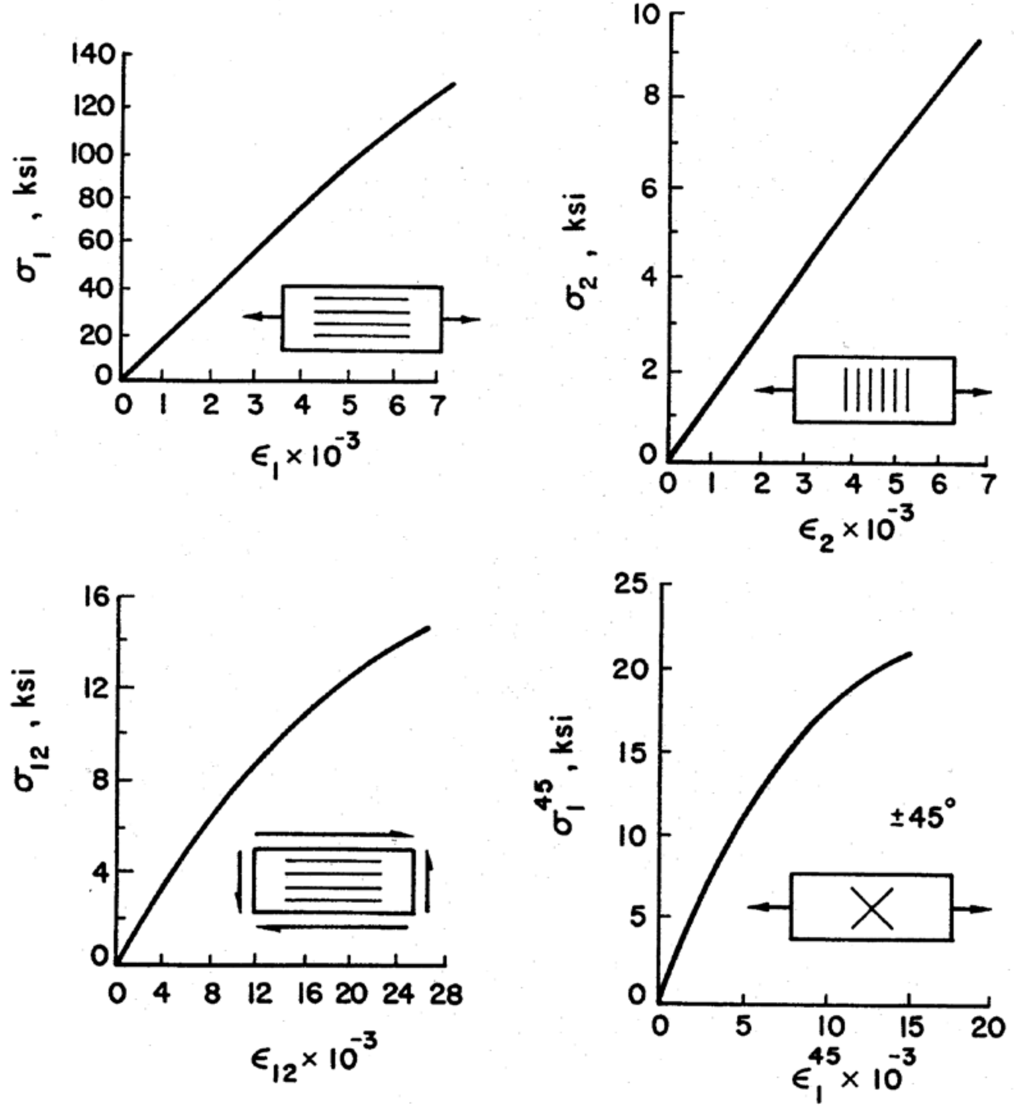




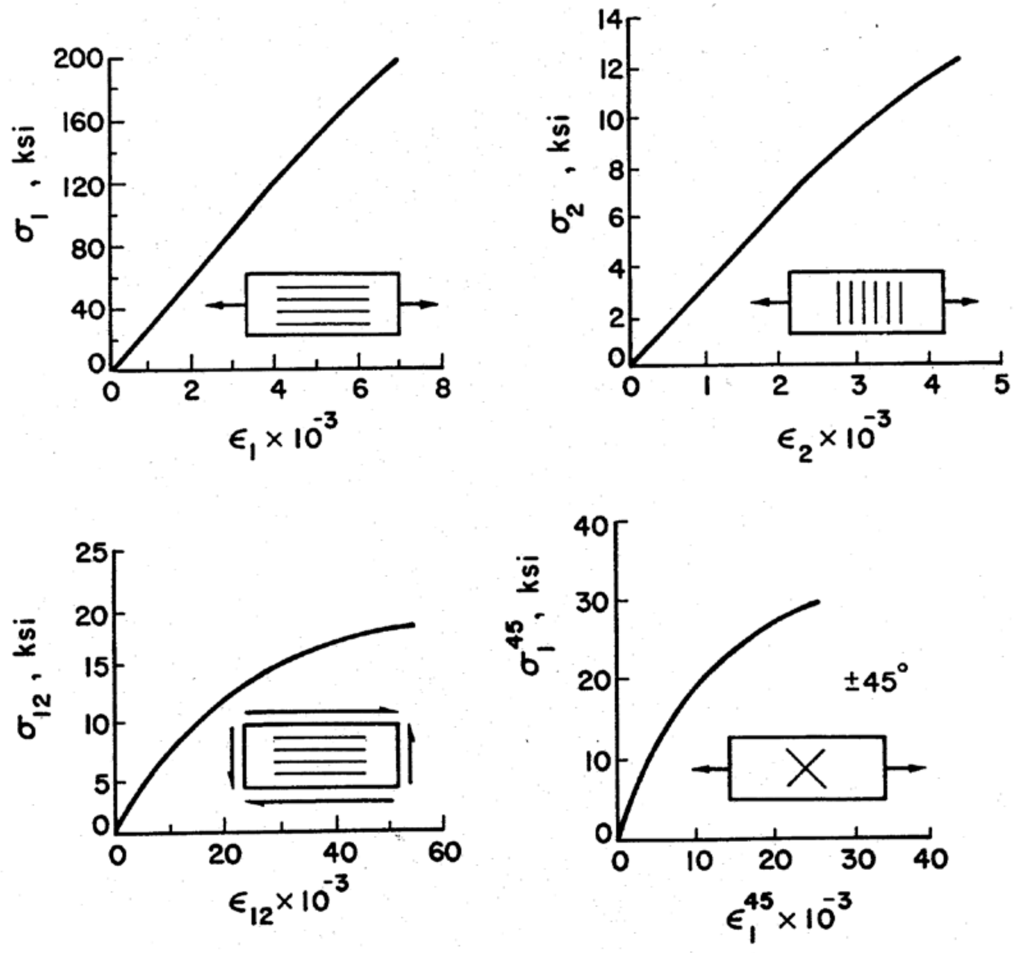
Stress-Strain Curves for Boron/Aluminum



Stress-Strain Curves for Graphite-Epoxy



Stress-Strain Curves for Boron-Epoxy



Finite Element Models of Nonlinear Elasticity

Nonlinear Constitutive Equation

$$\sigma_{xx} = E\ F(\varepsilon_{xx})$$

Virtual Work Statement

$$0 = \int_A \int_{x_a}^{x_b} \sigma_{xx}\delta\varepsilon_{xx}\ dx dA - \int_{x_a}^{x_b} f\delta u\ dx - P_1^e \delta u(x_a) - P_2^e \delta u(x_b)$$

$$= \int_{x_a}^{x_b} \left[EA\ F(\varepsilon_{xx})\delta\varepsilon_{xx} - f\delta u\right] dx - P_1^e \delta u(x_a) - P_2^e \delta u(x_b)$$

Finite Element Models of Nonlinear Elasticity (continued)

Finite Element Model

$$R_i^e = \int_{x_a}^{x_b} \left[EA\ F(\varepsilon_{xx}) \frac{d\psi_i^e}{dx} - f\psi_i \right] dx - P_i^e$$

$$K_{ij}^e = \frac{\partial R_i^e}{\partial u_j^e} = EA \int_{x_a}^{x_b} \frac{\partial F}{\partial \varepsilon_{xx}} \frac{\partial \varepsilon_{xx}}{\partial u_j^e} \frac{d\psi_i^e}{dx}\ dx$$

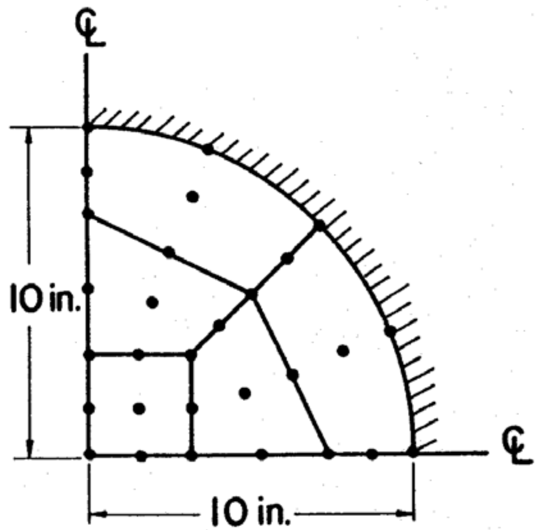
$$= EA \int_{x_a}^{x_b} \left(\frac{\partial F}{\partial \varepsilon_{xx}} \right) \frac{d\psi_i^e}{dx} \frac{d\psi_j^e}{dx}\ dx$$

Ramberg-Osgood Model

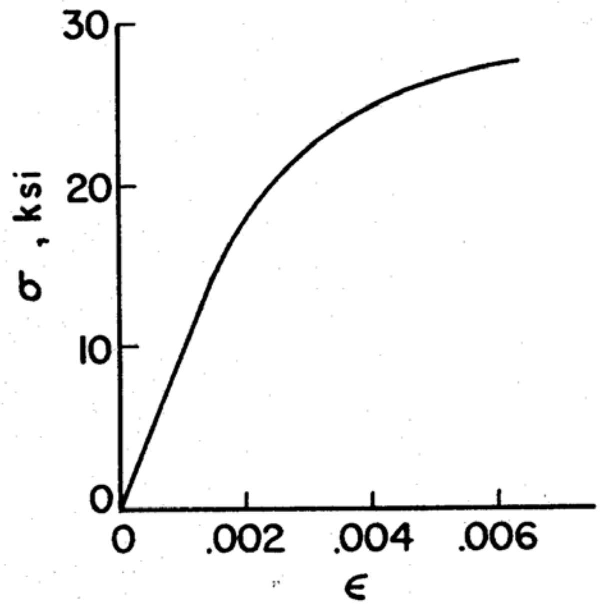
$$F(\varepsilon_{xx}) = (\varepsilon_{xx})^n$$

Numerical Examples

MATERIALLY NONLINEAR ANALYSIS

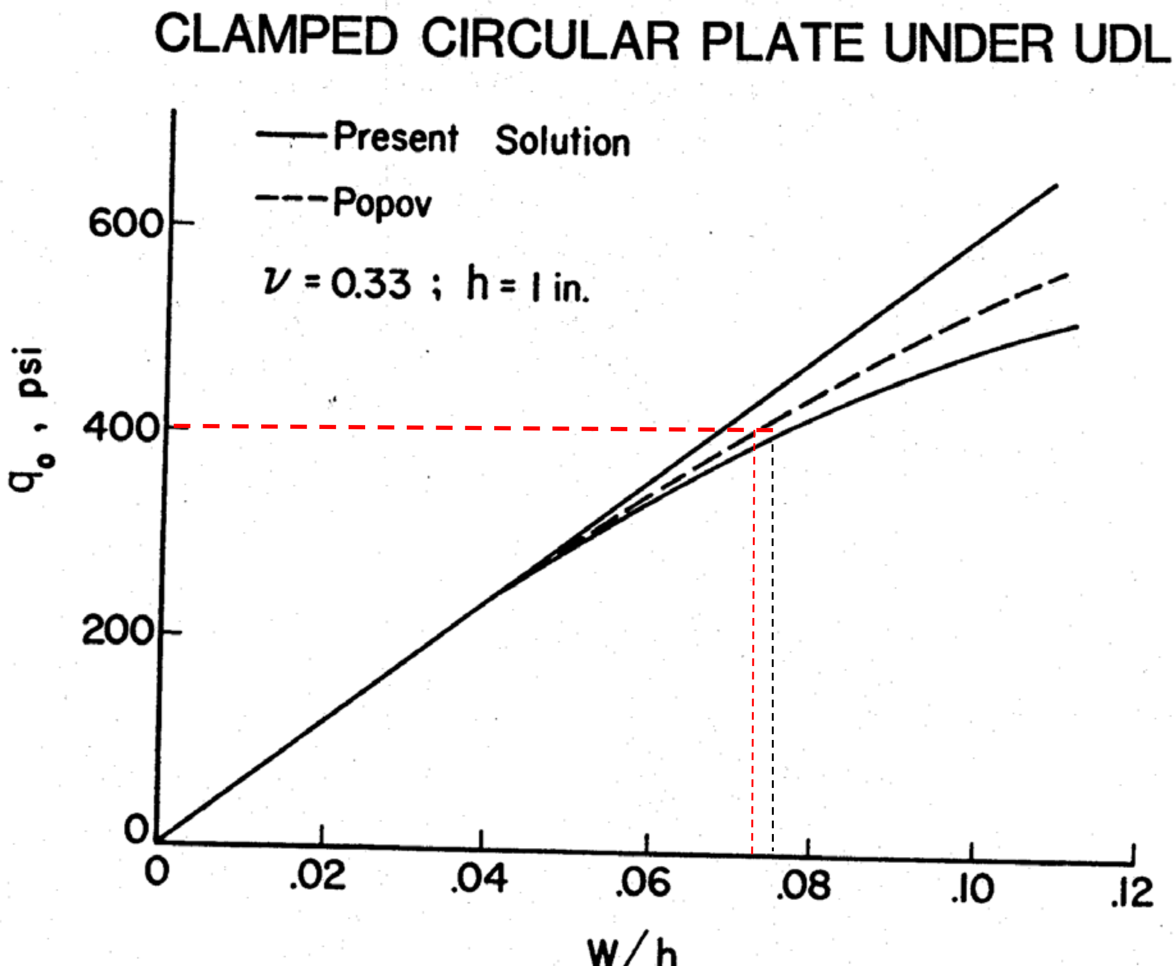


(a) 9-Node Plate Model



(b) Uniaxial Stress-strain Diagram of Plate Material

Numerical Examples



Load-Displacement Curve

Small Deformation Theory of Plasticity

The theory of plasticity deals with an analytical description of the stress-strain relations of a deformed body after a part or all of the body has yielded.

The stress-strain relations must contain:

1. The elastic stress-strain relations.

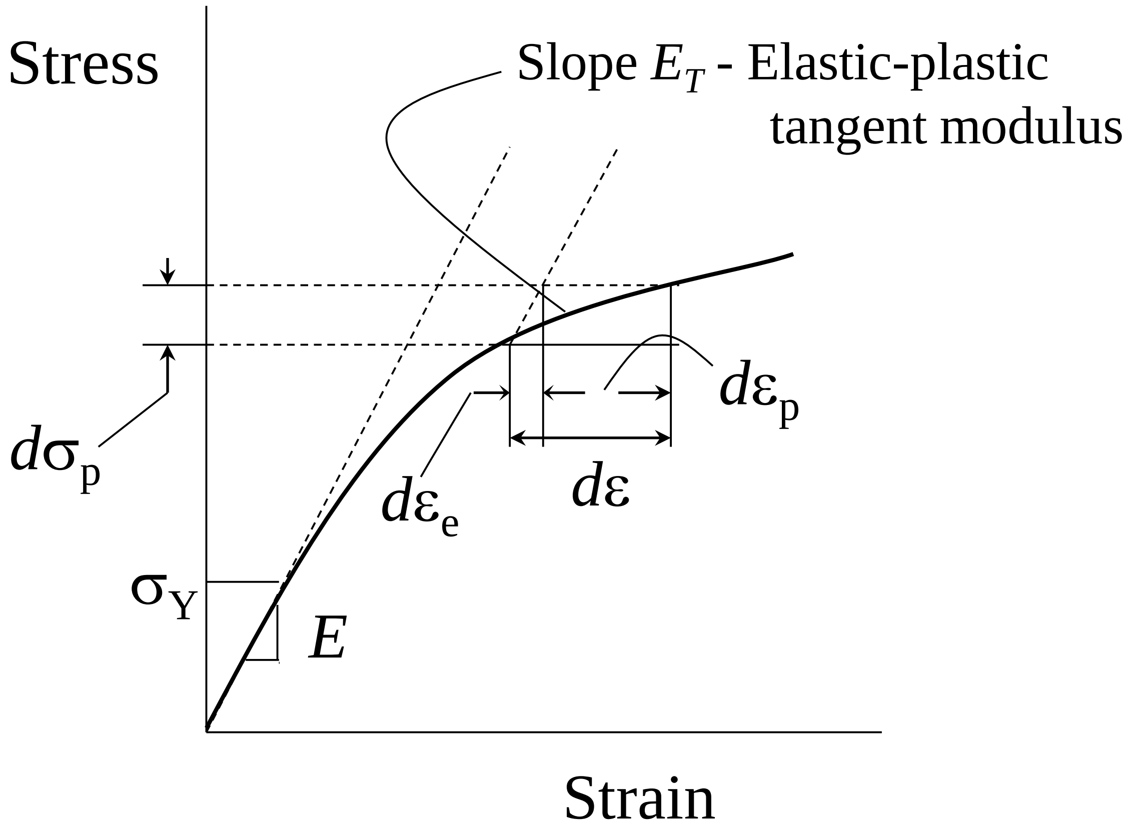
2. The stress condition (or *yield criterion*) which indicates onset of yielding.

3. The stress-strain or stress-strain increment relations after the onset of plastic flow.

Small Deformation Theory of Plasticity

$\sigma_{ij} < \sigma_Y$ linear elastic behavior

$\sigma_{ij} \geq \sigma_Y$ plastic deformation (not recoverable)



Small Deformation Theory of Plasticity (continued)

General Yield Criterion

$$F(\sigma_{ij}, \kappa) = 0$$

$$F(J_2', J_3', \kappa) = 0, \quad J_2' = \frac{1}{2}\sigma'_{ij}\sigma'_{ij}, \quad J_3' = \frac{1}{3}\sigma'_{ij}\sigma'_{jk}\sigma'_{k\ell}$$

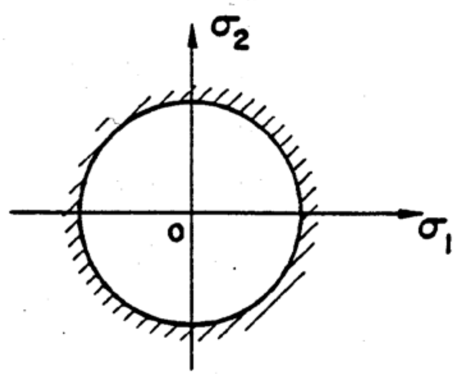
The Tresca yield criterion:

$$F = 2\bar{\sigma}\cos\theta - Y(\kappa) = 0, \quad \bar{\sigma} = \sqrt{J_2'}$$

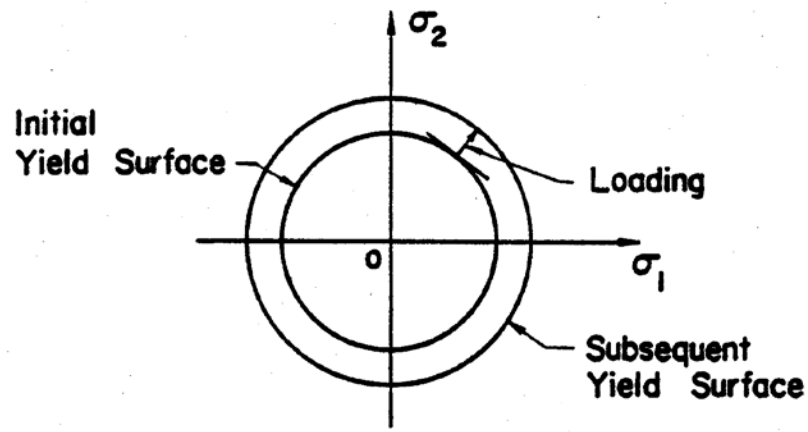
The Huber-von Mises yield criterion:

$$F = \sqrt{3\bar{\sigma}} - Y(\kappa) = 0$$

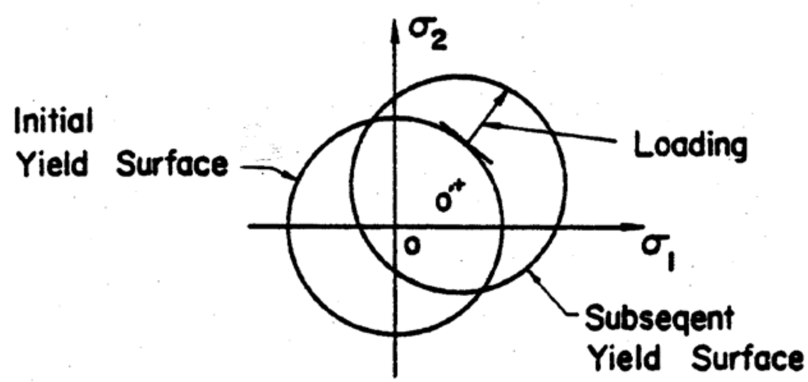
Mathematical Models of the Strain Hardening Behavior



(a) Perfectly Plastic



(b) Isotropic Strain Hardening



(c) Kinematic Strain Hardening

Small Deformation Theory of Plasticity (continued)

$$d\varepsilon = d\varepsilon^e + d\varepsilon^p, \quad d\varepsilon^e = \frac{d\sigma}{E}, \quad \frac{d\sigma}{d\varepsilon} = E_T$$

$$H = \frac{d\sigma}{d\varepsilon^p} = \frac{\frac{d\sigma}{d\varepsilon}}{1 - \frac{d\varepsilon^e}{d\varepsilon}} = \frac{E_T}{1 - \frac{E_T}{E}}$$

Finite Element Formulation

$$[K^e] = \int_{x_a}^{x_b} [B]^T [D^e][B] dx$$

$$du = h_e d\varepsilon_{xx} = h_e \left(d\varepsilon^e + d\varepsilon^p\right)$$
$$dF = A d\sigma = A_e H d\varepsilon^p$$

$$E^{ep} = \frac{dF}{du} = \frac{A_e H d\varepsilon^p}{h_e \left(d\varepsilon^e + d\varepsilon^p\right)} = \frac{E A_e}{h_e}\left[1 - \frac{E}{(E+H)}\right]$$

$$[K^{ep}] = \int_{x_a}^{x_b} [B]^T [D^{ep}][B] dx$$

Small Deformation Theory of Plasticity (continued)

$$\{d\varepsilon\} = \{d\varepsilon^e\} + \{d\varepsilon^p\}$$

$$\{d\varepsilon^e\} = [D^e]^{-1}\{d\sigma\}, \quad \{d\varepsilon^p\} = d\lambda \left\{\frac{df}{d\{\sigma\}}\right\}$$

$$\begin{aligned}\{d\varepsilon^e\} &= \{d\varepsilon\} - \{d\varepsilon^p\} \\ &= \{d\varepsilon\} - d\lambda \left\{\frac{\partial f}{\partial\{\sigma\}}\right\}\end{aligned}$$

$$\{d\sigma\} = [D^e]\left(\{d\varepsilon\} - d\lambda \left\{\frac{\partial f}{\partial\{\sigma\}}\right\}\right)$$

$$0 = f(\{\sigma\}, \kappa) \quad \rightarrow \quad 0 = df = \left\{\frac{\partial f}{\partial\{\sigma\}}\right\}^{\mathrm{T}} \{d\sigma\} - A d\lambda$$

$$0 = \left\{\frac{\partial f}{\partial\{\sigma\}}\right\}^{\mathrm{T}} [D^e]\left(\{d\varepsilon\} - d\lambda \frac{\partial f}{\partial\{\sigma\}}\right) - A d\lambda$$

Small Deformation Theory of Plasticity (continued)

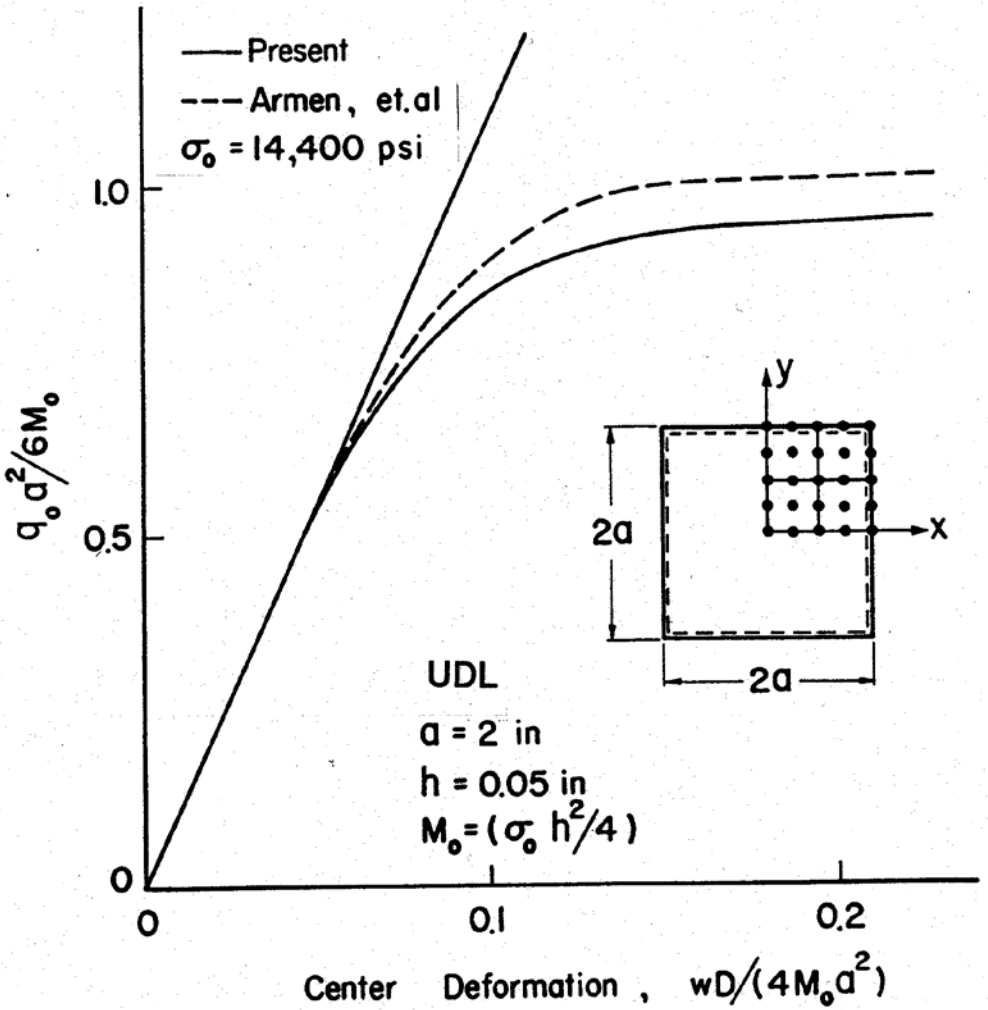
$$d\lambda = \frac{\{\frac{\partial f}{\partial\{\sigma\}}\}^{\mathrm{T}}[D^e]\{d\varepsilon\}}{A + \{\frac{\partial f}{\partial\{\sigma\}}\}[D^e]\{\frac{\partial f}{\partial\{\sigma\}}\}}$$

$$A = H = \frac{d\bar{\sigma}}{d\bar{\varepsilon}^p} = \frac{E_T}{1 - E_T/E}$$

$$\{d\sigma\} = [D^{ep}]\{d\varepsilon\}$$

$$[D^{ep}] = [D^e] - \frac{[D^e]\frac{\partial f}{\partial\{\sigma\}}\{\frac{\partial f}{\partial\{\sigma\}}\}^T[D^e]}{H + \{\frac{\partial f}{\partial\{\sigma\}}\}^T[D^e]\{\frac{\partial f}{\partial\{\sigma\}}\}}$$

Numerical Examples



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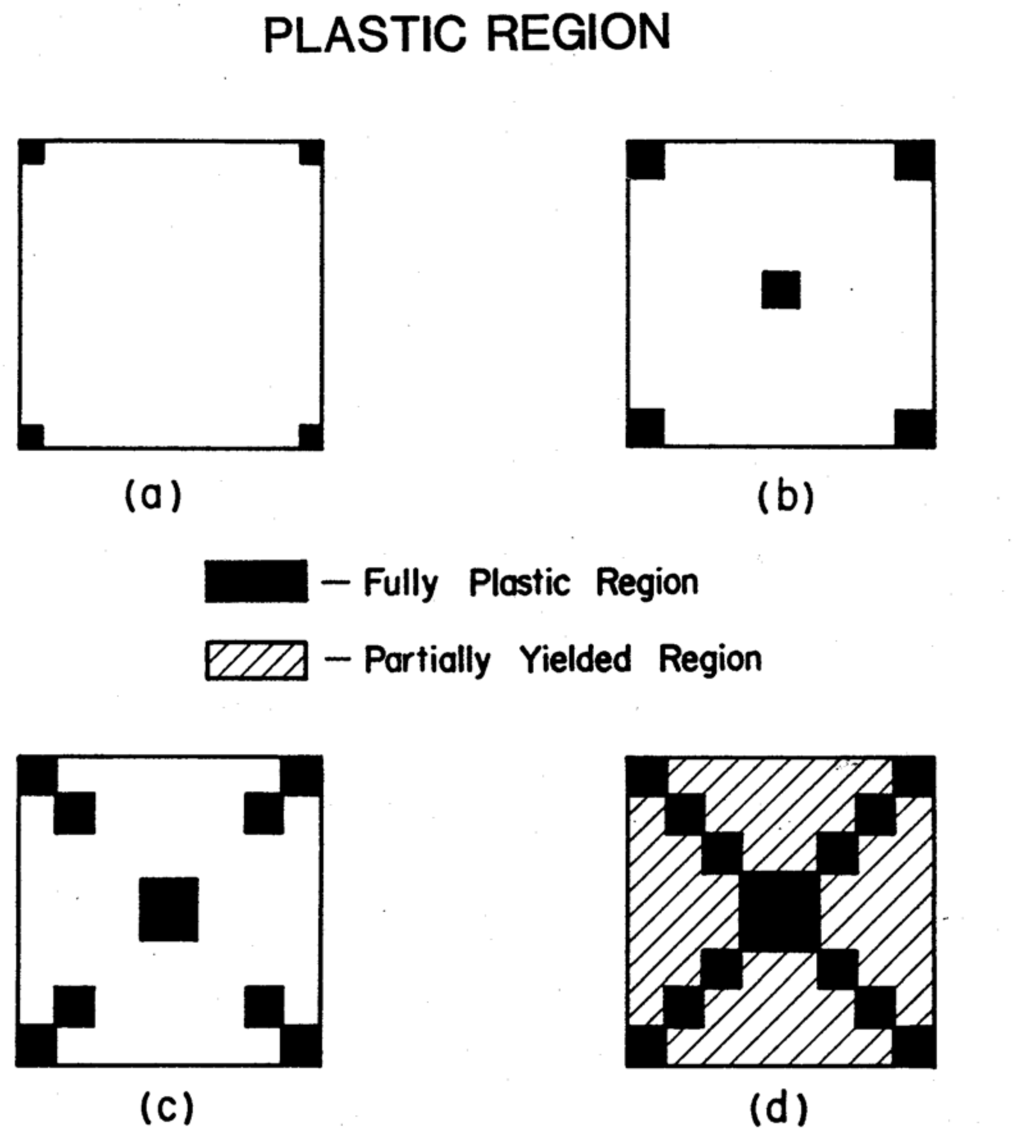
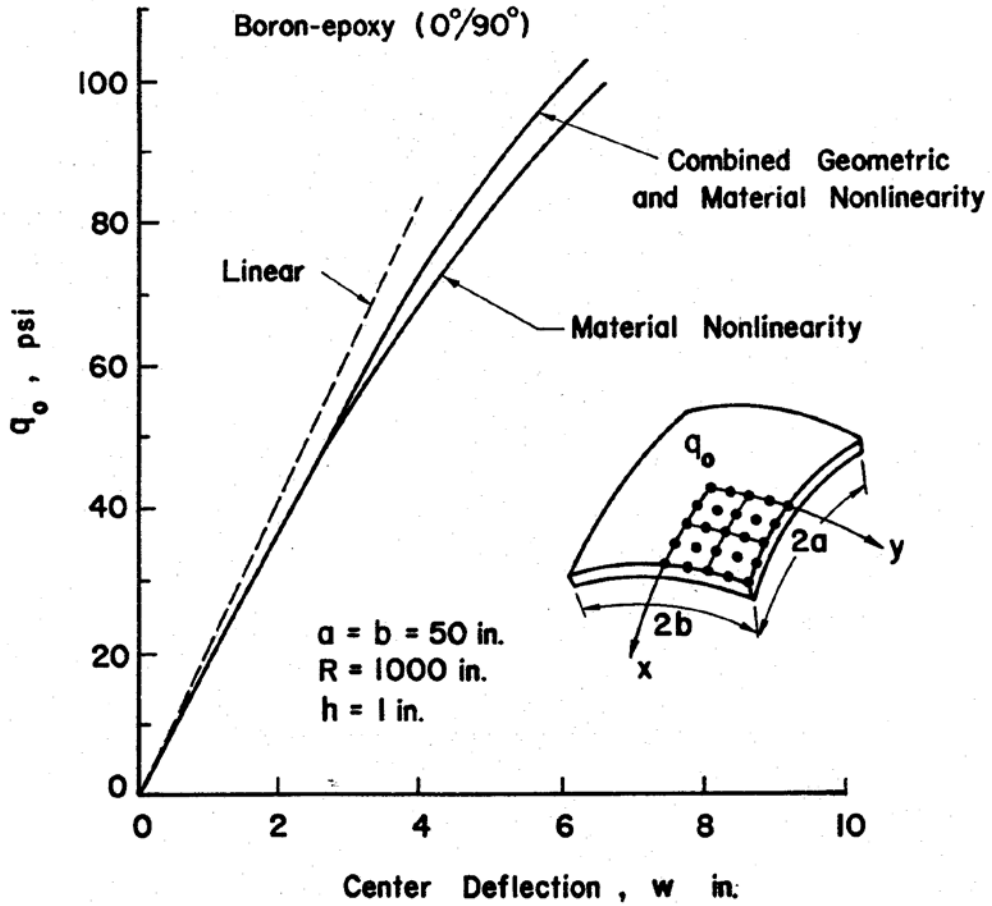


Numerical Examples

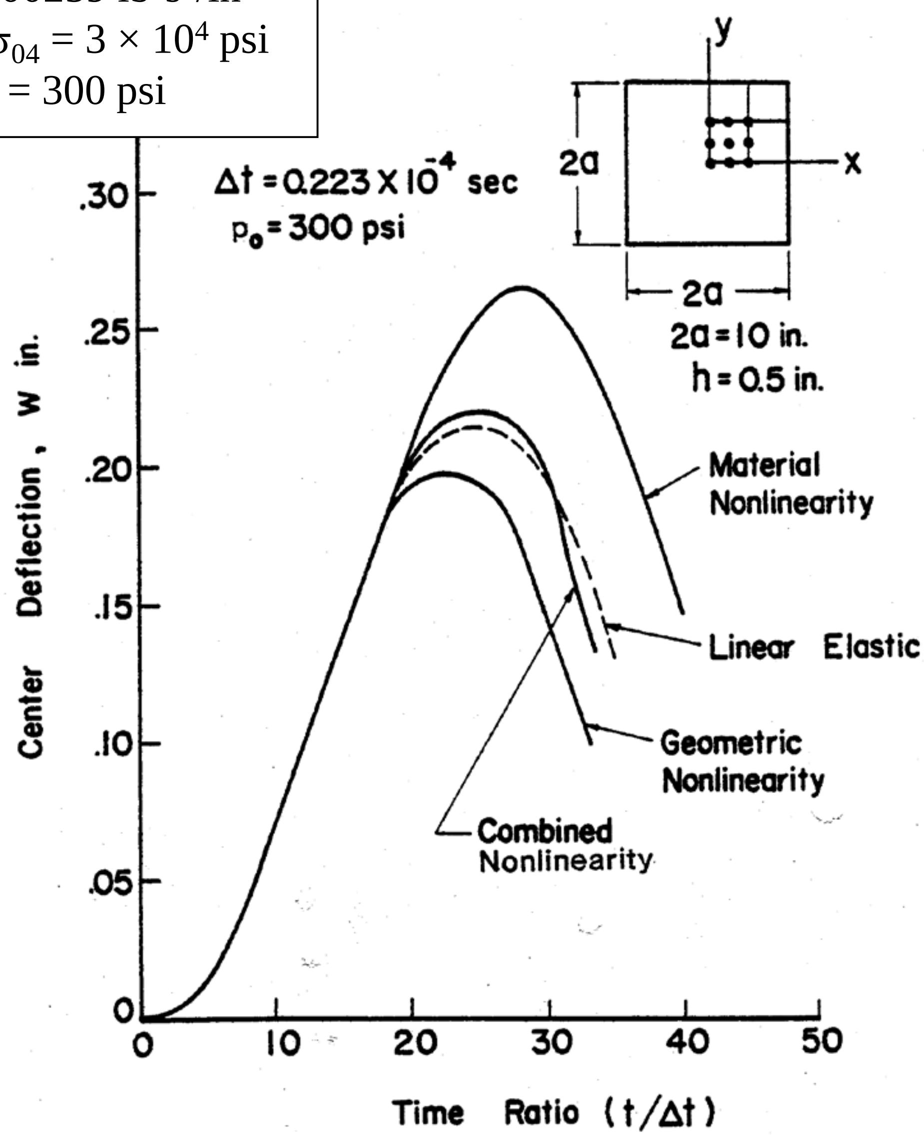


Simply Supported Isotropic Plate Under UDL
(Combined Material and Geometric Nonlinearity)

$E = 10^6$ psi., $G = 3.846\ E$, $\nu = 0.3$, $\rho = 0.000259$ lb-s^2/in^4

$\sigma_i = \sigma_{0i} + E_{pi}\ \varepsilon^p$ (i=1,2,4,5,6); $\sigma_{01} = \sigma_{02} = \sigma_{04} = 3 \times 10^4$ psi

$\sigma_{04} = \sigma_{05} = \sigma_{06} = 1.372 \times 10^4$ psi, $p_0 = 300$ psi



Summary

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