COUPLED FLUID FLOW AND HEAT TRANSFER PROBLEMS

- Governing equations
- Coupled fluid flow and heat transfer
- Numerical Examples

Governing Equations of Flows of Viscous Incompressible Fluids

Conservation of Mass

$$\frac{D\rho}{Dt} + \rho\nabla \cdot \mathbf{u} = 0, \quad \frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla$$

$$\nabla \cdot \mathbf{u} = 0$$

Conservation of Momentum

$$\rho\frac{D\mathbf{u}}{Dt} = \nabla \cdot \sigma + \rho\mathbf{f}$$

Governing Equations (continued)

Conservation of Energy

$$\rho C_v \frac{DT}{Dt} = -\nabla \cdot \mathbf{q} + Q + \Phi$$

$$\Phi = \tau : \mathbf{D}$$

Constitutive Equations

$$\rho = \rho_0[1 - \beta(T - T_0)]$$

$$\sigma = -P\mathbf{I} + \tau, \quad \tau = \lambda(\mathrm{tr}\ \mathbf{D})\mathbf{I} + 2\mu\mathbf{D}$$

$$\mathbf{D} = \frac{1}{2}\left[(\nabla\mathbf{u}) + (\nabla\mathbf{u})^T\right]$$

Conservation of Energy

$$\rho\varepsilon - \nabla\cdot\mathbf{q} + \nabla\mathbf{v}:\boldsymbol{\sigma} + \rho\left[\frac{\partial e}{\partial t} + \mathbf{v}\cdot\nabla e\right] = 0$$

internal heat generation (W/m^3)

heat flux (W/m^2)

stress power

total time rate of change of internal energy

Steady-state: $\frac{\partial e}{\partial t} = 0$

Solid medium: $\mathbf{v}.\nabla e = 0$

Always known (e.g., nuclear fission or chemical reactions)

Known in terms the temperature via Fourier's Law

$$\mathbf{q} = -k\nabla T$$

Thermal conductivity [W/(m.ºC)]

Highly nonlinear, and often neglected

Governing Equations (component form)

$$\frac{\partial u_i}{\partial x_i} = 0$$

$$\rho_0 \left(\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) = \frac{\partial}{\partial x_j} \left[-P\delta_{ij} + \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right]$$

$$- \rho_0 g_i \beta (T - T_0)$$

$$\rho_0 C \left(\frac{\partial T}{\partial t} + u_j \frac{\partial T}{\partial x_j} \right) = \frac{\partial}{\partial x_i} \left(k \frac{\partial T}{\partial x_i} \right) + Q + 2\mu D_{ij} D_{ij}$$

$$\rho_s C_s \frac{\partial T}{\partial t} = \frac{\partial}{\partial x_i} \left(k_s \frac{\partial T}{\partial x_i} \right) + Q_s$$

$$D_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \quad \sigma_{ij} = \tau_{ij} - P\delta_{ij}, \quad \tau_{ij} = 2\mu D_{ij}$$

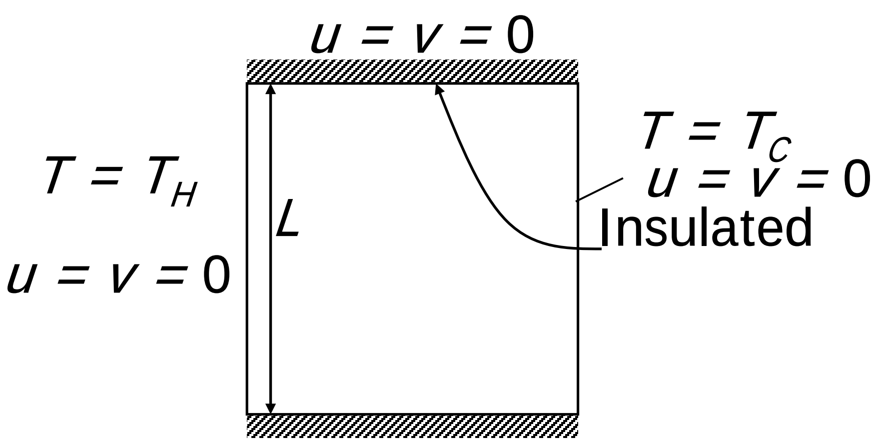
FINITE ELEMENT MODEL OF THE ENERGY EQUATION
(Coupled Heat Transfer and Fluid Flow

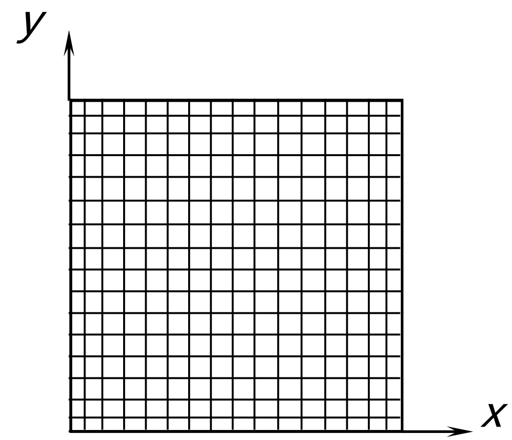
$$\rho_0 C\left(\frac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla T\right) = k\nabla^2 T + Q$$

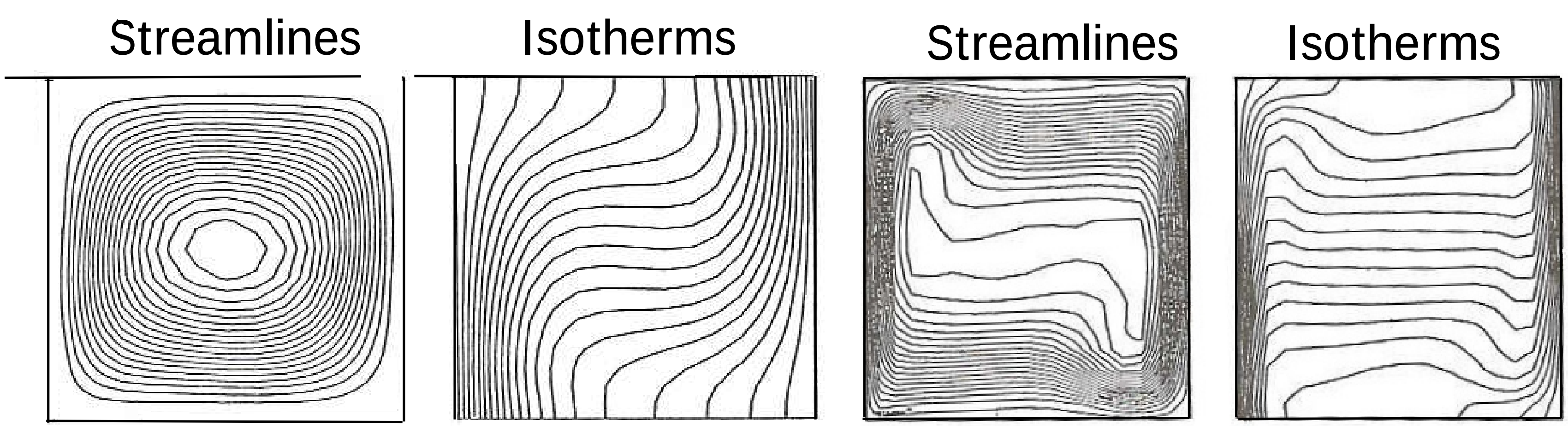
$$[M]\{\dot{T}\} + [K_{\mathbf{T}}(\{\mathbf{v}\})]\{T\} = \{Q\}$$

$$[M]\{\dot{\Delta}\} + [K_v]\{\Delta\} = \{F(T)\}$$

Numerical Examples



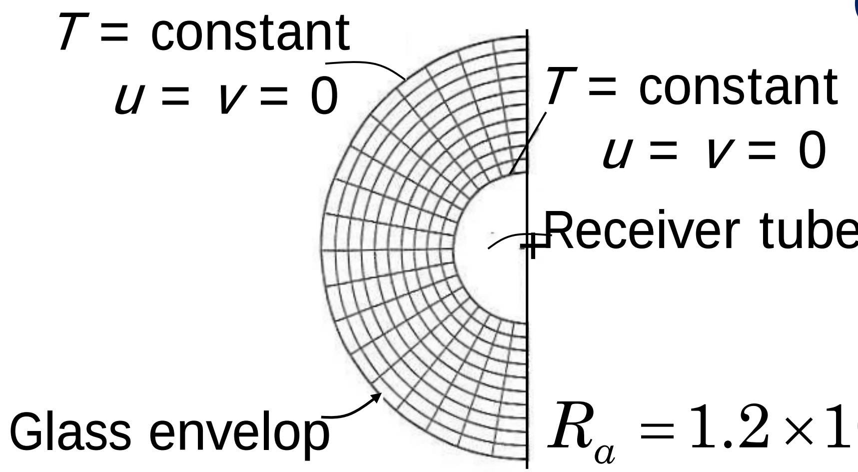




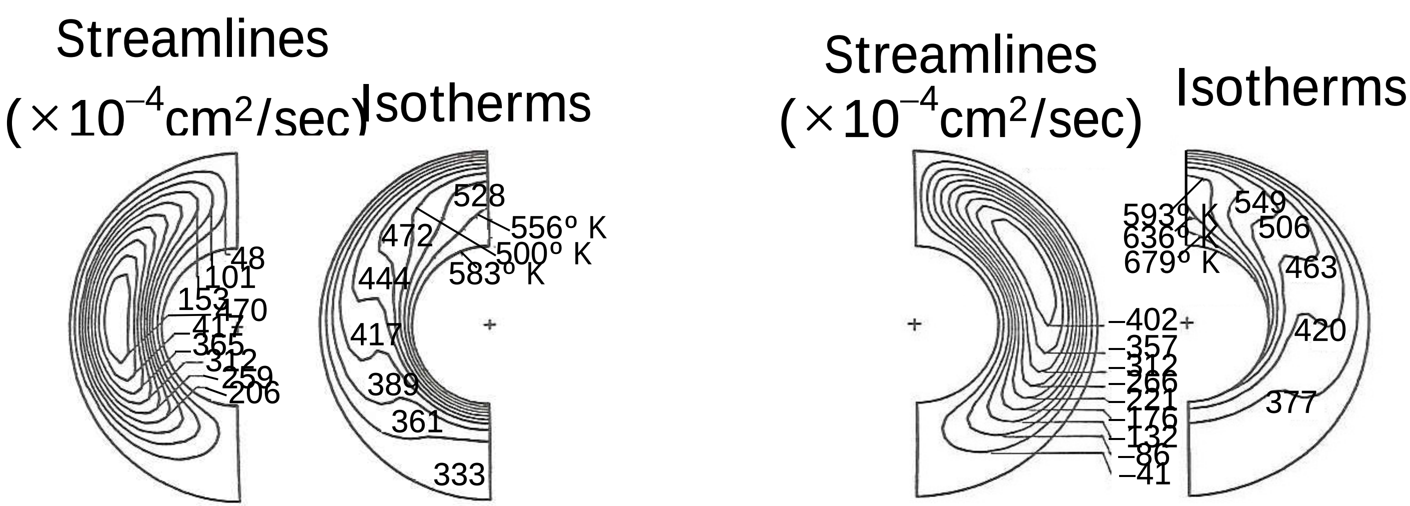
$R_a = 10^4, \; P_r = 0.71$ $\qquad$ $R_a = 10^6, \; P_r = 0.71$

Coupled Fluid Flow and Heat Transfer

(continued)



$$R_a = 1.2 \times 10^4, \; P_r = 0.71$$



(a) Uniform temperature

(b) Hot on top

Coupled Fluid Flow and Heat Transfer
(continued)

$$R_a = 1.2 \times 10^4, \ P_r = 0.71$$