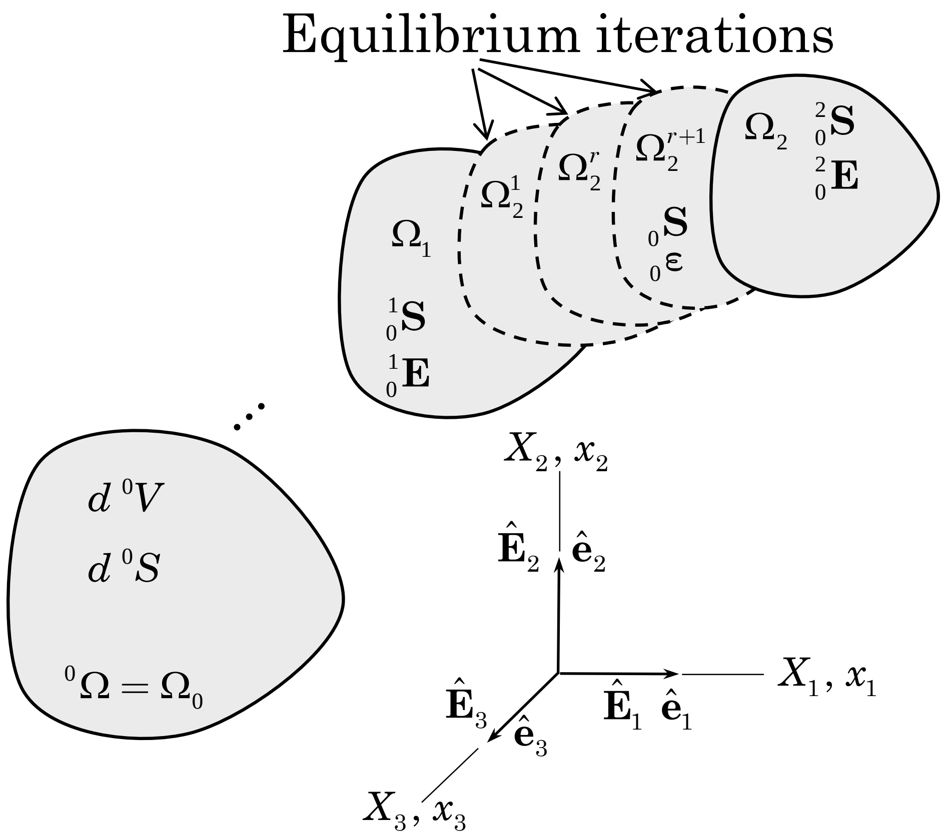
CONTINUUM SHELL ELEMENT AND POSTBUCKLING ANALYSIS

- Description of continuum shell element (principle of virtual displacements)
- Post-buckling analysis of composite panels and comparison with experimental results

CONTINUUM SHELL ELEMENT



CONTINUUM SHELL ELEMENT

Principle of virtual displacement applied to the configuration to be determined

$$\int_{{}^{t+\Delta t}V} {}^{t+\Delta t}\sigma_{ij}\ \delta_{t+\Delta t}e_{ij}\ {}^{t+\Delta t}dV = {}^{t+\Delta t}R$$

Use energy conjugacy to write all integrals on the reference configuration

$$\int_{{}^{t+\Delta t}V} {}^{t+\Delta t}t_{ij}\ \delta({}_{t+\Delta t}e_{ij})\ {}^{t+\Delta t}dV = \int_{{}^{0}V} {}^{t+\Delta t}_{0}S_{ij}\ \delta({}^{t+\Delta t}_{0}\varepsilon_{ij})\ {}^{0}dV$$

$$\int_{{}^{0}V} {}^{t+\Delta t}_{0}S_{ij}\ \delta({}^{t+\Delta t}_{0}\varepsilon_{ij})\ {}^{0}dV = {}^{t+\Delta t}R$$

$${}^{t+\Delta t}R = \int_{{}^{0}A} {}^{t+\Delta t}_{0}t_{k}\ \delta u_{k}\ {}^{0}dA + \int_{{}^{0}V} {}^{0}\rho\ {}^{t+\Delta t}_{0}f_{k}\ \delta u_{k}\ {}^{0}dV$$

CONTINUUM SHELL ELEMENT

Decompositions

$$ {}^{t+\Delta t}_{\ 0}S_{ij} = {}^{t}_{0}S_{ij} + {}_{0}S_{ij} \qquad {}_{0}\varepsilon_{ij} = {}_{0}\ell_{ij} + {}_{o}\eta_{ij} $$

$$ {}^{t+\Delta t}_{\ 0}\varepsilon_{ij} = {}^{t}_{0}\varepsilon_{ij} + {}_{0}\varepsilon_{ij} \qquad {}_{0}\eta_{ij} = \frac{1}{2}\ {}_{0}u_{k,i}\ {}_{0}u_{k,j} $$

$$ {}_{0}\ell_{ij} = \frac{1}{2}\left({}_{0}u_{i,j} + {}_{0}u_{j,i} + {}^{t}_{0}u_{k,i}\ {}_{0}u_{k,j} + {}_{0}u_{k,i}\ {}^{t}_{0}u_{k,j}\right) $$

$$ \delta\left({}^{t+\Delta t}_{\ 0}\varepsilon_{ij}\right) = \delta\left({}^{t}_{0}\varepsilon_{ij}\right) + \delta\left({}_{0}\varepsilon_{ij}\right) = \delta\left({}_{0}\varepsilon_{ij}\right) $$

$$ {}_{0}S_{ij} = {}_{0}C_{ijrs}\ {}_{0}\varepsilon_{rs}, \quad {}_{0}S_{ij} = {}_{0}C_{ijrs}\ {}_{0}\ell_{rs} $$

Final virtual work statement

$$ \int_{{}^{0}V} {}_{0}C_{ijrs}\ {}_{0}\ell_{rs}\ \delta\left({}_{0}\varepsilon_{ij}\right)\ {}^{0}dV + \int_{{}^{0}V} {}^{t}_{0}S_{ij}\ \delta\left({}_{0}\eta_{ij}\right)\ {}^{0}dV $$

$$ = {}^{t+\Delta t}R - \int_{{}^{0}V} {}^{t}_{0}S_{ij}\ \delta\left({}_{0}\ell_{ij}\right)\ {}^{0}dV $$

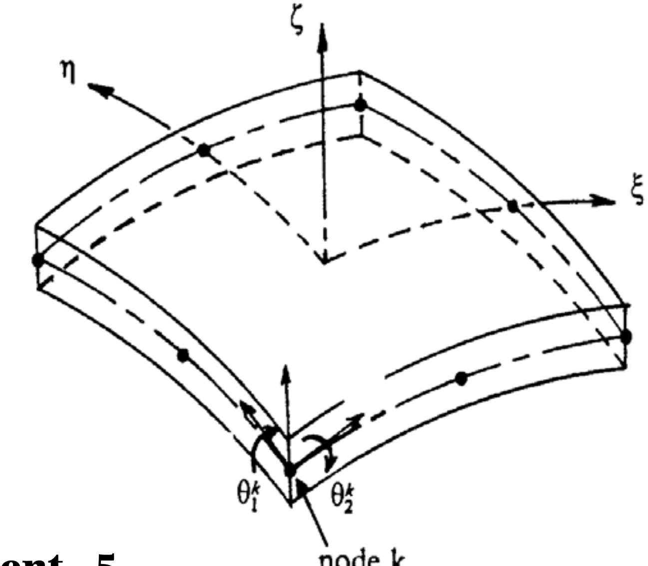
CONTINUUM SHELL ELEMENT
(continued)

Assumed Kinematics of the Shell Element

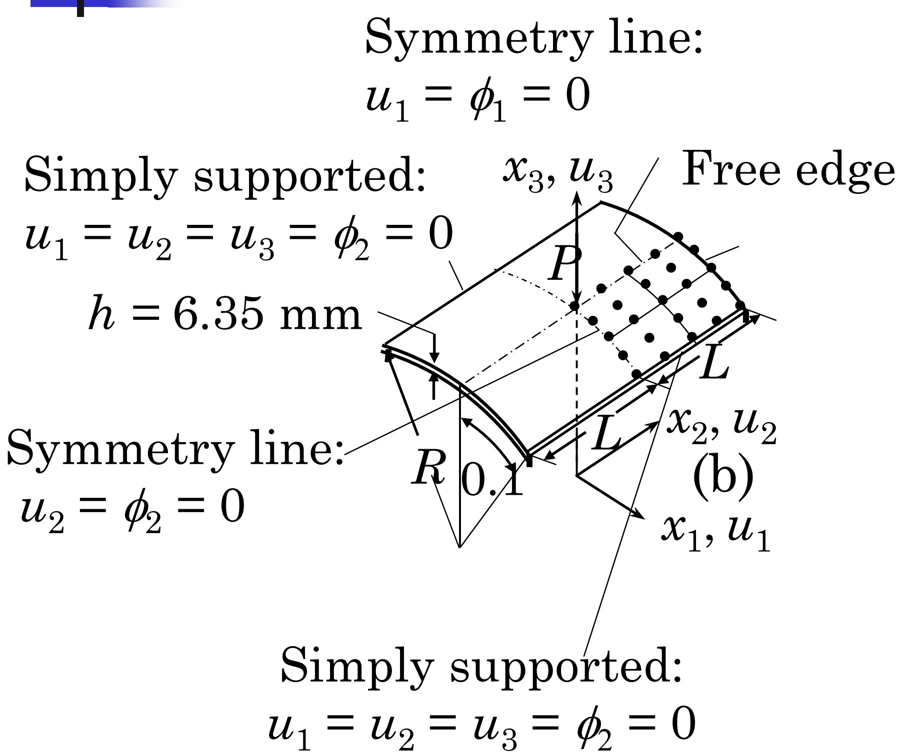
$$ {}^t x_i = \sum_{k=1}^{n} \psi_k(\xi,\eta)[{}^t x_i^k + \frac{\zeta}{2} h_k \, {}^t e_{3i}^k] $$

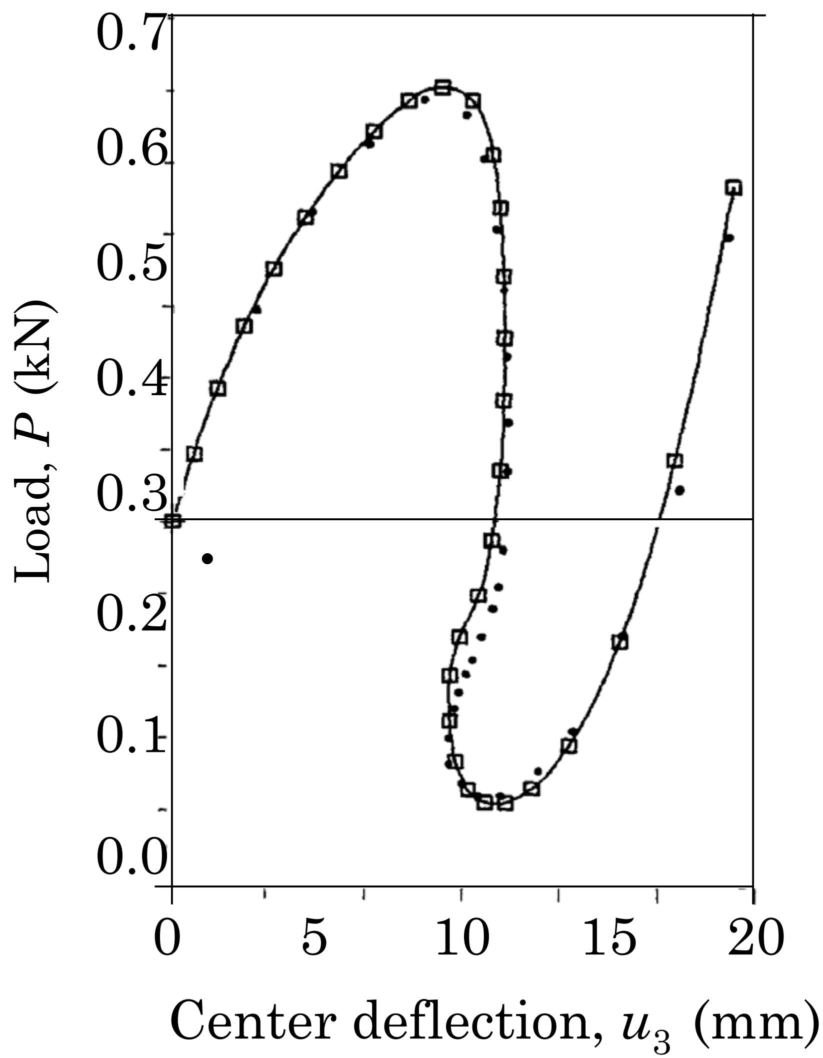
$$ {}^t u_i = {}^t x_i - {}^0 x_1 = \sum_{k=1}^{n} \psi_k(\xi,\eta)[{}^t u_k^k + \frac{\zeta}{2} h_k ({}^t e_{3i}^k - {}^0 e_{3i}^k)] $$

$$ u_i = {}^{t+\Delta t} u_i - {}^t u_i = \sum_{k=1}^{n} \psi_k(\xi,\eta)[u_i^k + \frac{\zeta}{2} h_k ({}^{t+\Delta t} e_{3i}^k - {}^t e_{3i}^k)] $$



A NUMERICAL EXAMPLE





$E = 3103$ N/mm^2, $\nu = 0.3$
$R = 2540$ mm, $L = 254$ mm

POSTBUCKLING AND FAILURE ANALYSIS - AN EXAMPLE

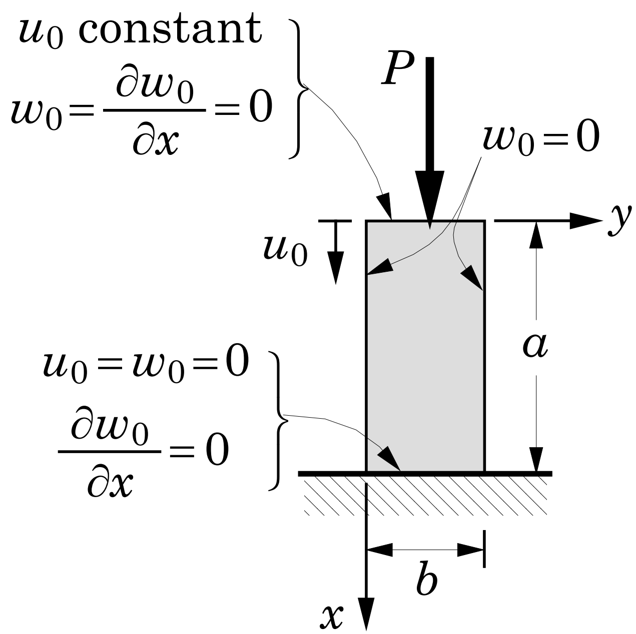
- Nonlinear FE analysis
- Comparison with experimental results of Starnes and Rouse
- Progressive failure analysis

a = 50.8 cm (20 in.), b = 17.8 cm (7 in.), h_k = 0.14 mm (0.0055 in.)

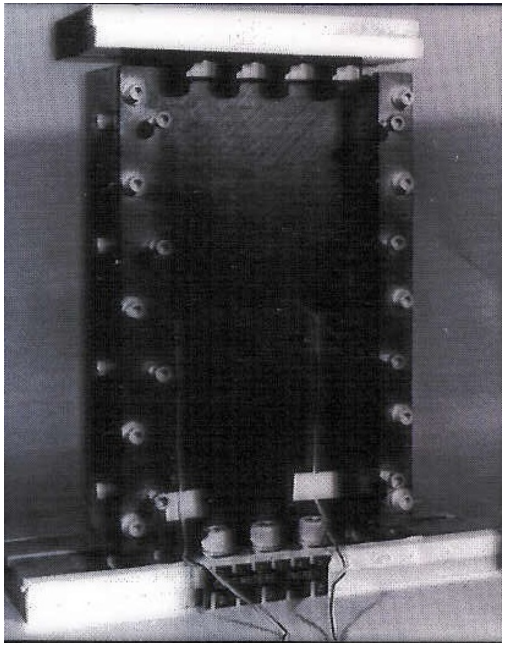
24-ply laminate: $[45/-45/0_2/45/-45/0_2/45/-45/0/90]_s$

E_1 = 131 Gpa (19,000 ksi), E_2 = 13 Gpa (1,890 ksi),

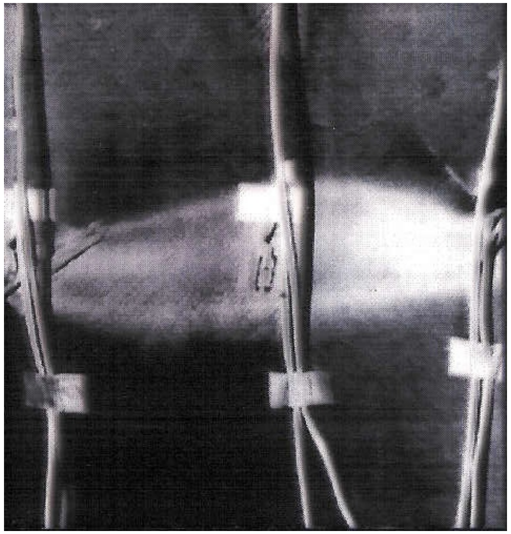
G_{12} = 6.4 Gpa (930 ksi), ν_{12} = 0.38 (graphite-epoxy)



Experimental Setup and Failure Region (Starnes & Rouse, NASA Langley)



(a) Typical panel with test fixture



(b) A transverse shear failure mode

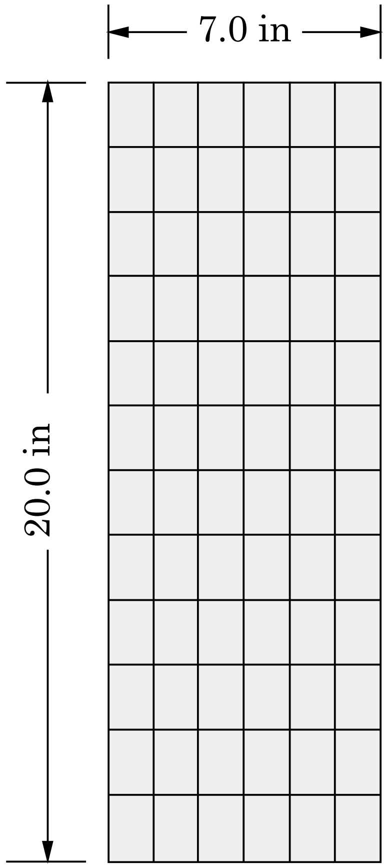
POST-BUCKLING OF A COMPOSITE PANEL UNDER IN-PLANE LOADING

Finite Element Models

- **Mesh has six elements per buckling mode half wave in each direction**

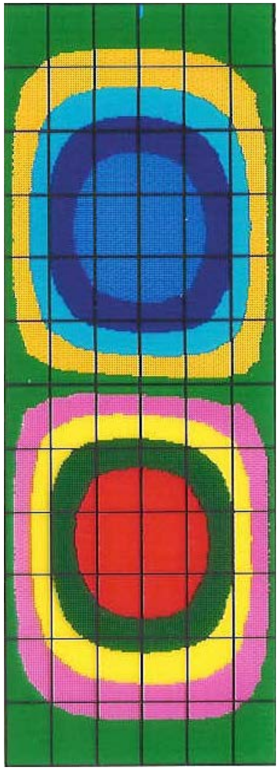
- **Elements used:**
  - **(1) Four-node C^1 -based flat shell element (STAGS)**
  - **(2) Nine-node continuum shell element (Chao & Reddy)**
  - **(3) For-node and nine-node ANS shell elements (Park & Stanley)**

- **All meshes have 72 elements (91 nodes for the meshes with four-node elements and 325 nodes for meshes with nine-node elements).**



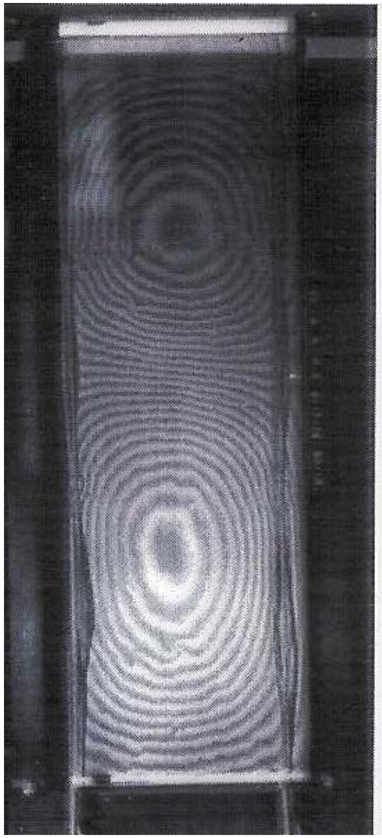
Comparison of the Experimental (Moire) and Analytical Out-of-Plane Deflection Patterns

Theoretical (FEM) Experimental

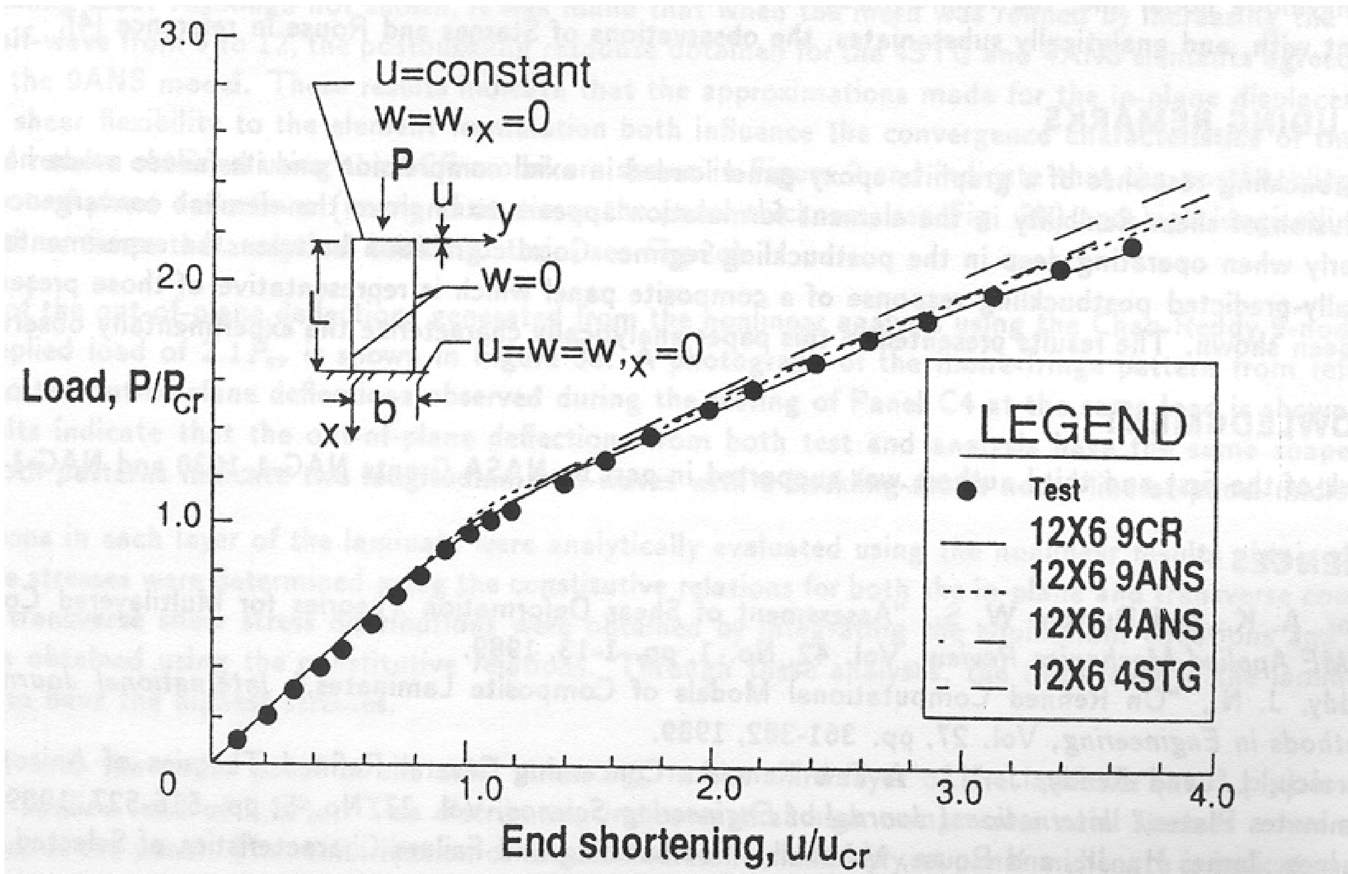


(a) Contour plot of the analytical results

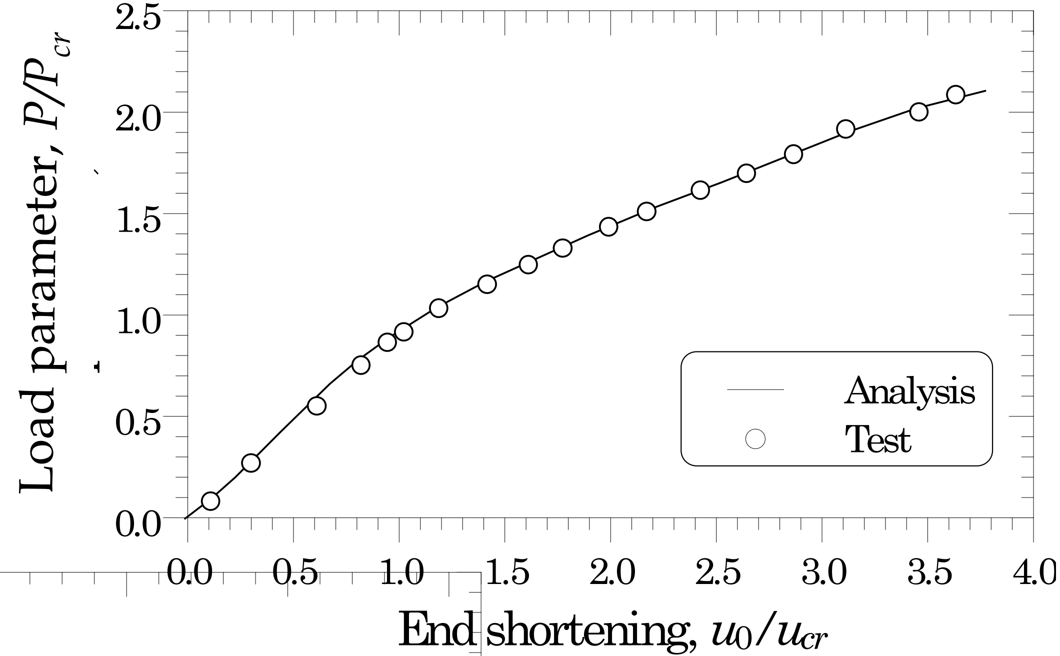
(b) Photograph of Moiré fringe pattern

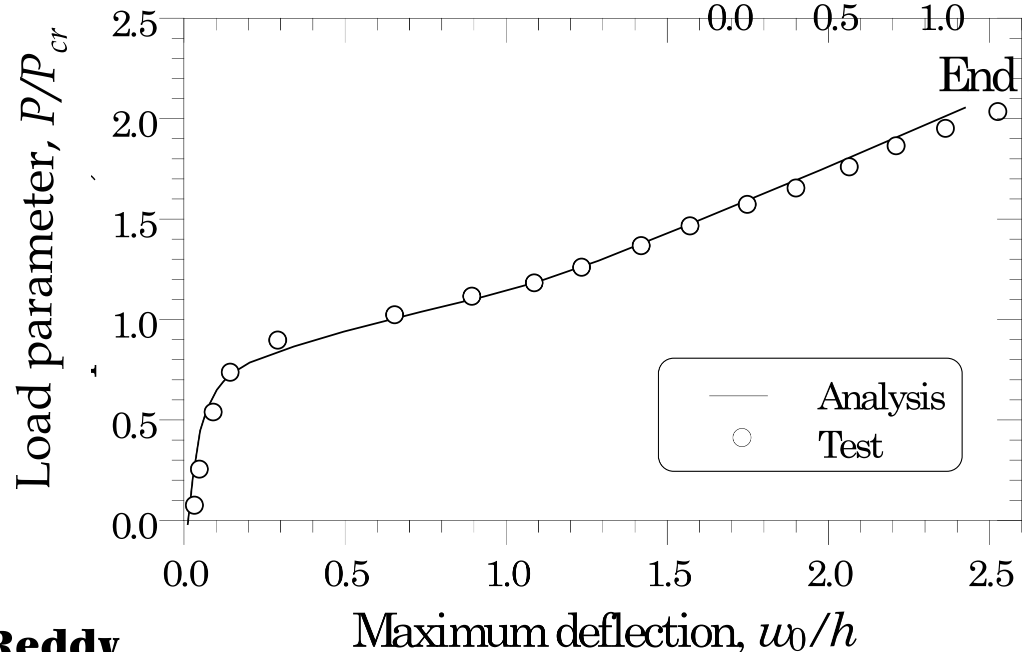


Comparison of the Experimental and Analytical Solutions for End Shortening

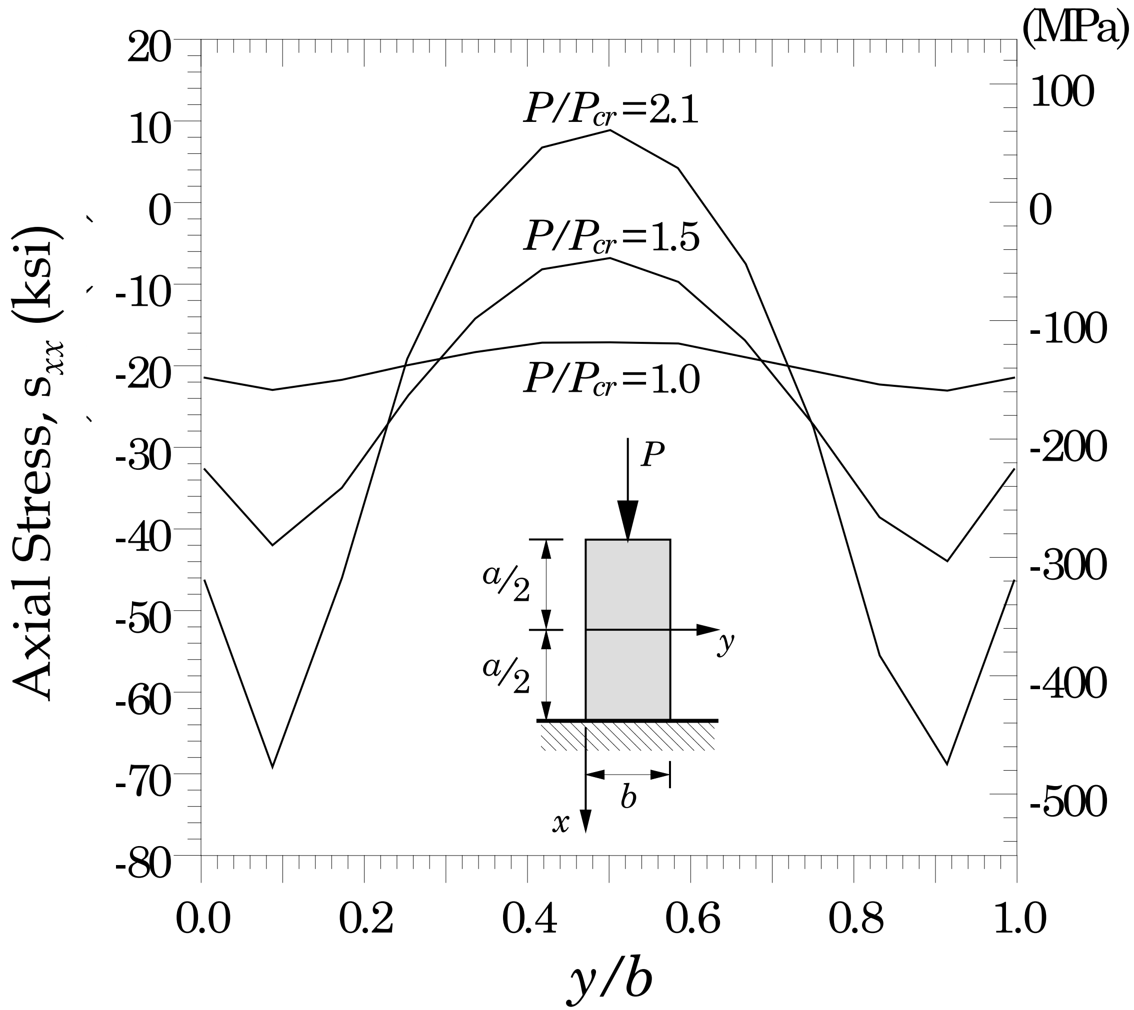


Postbuckling of a Composite Panel

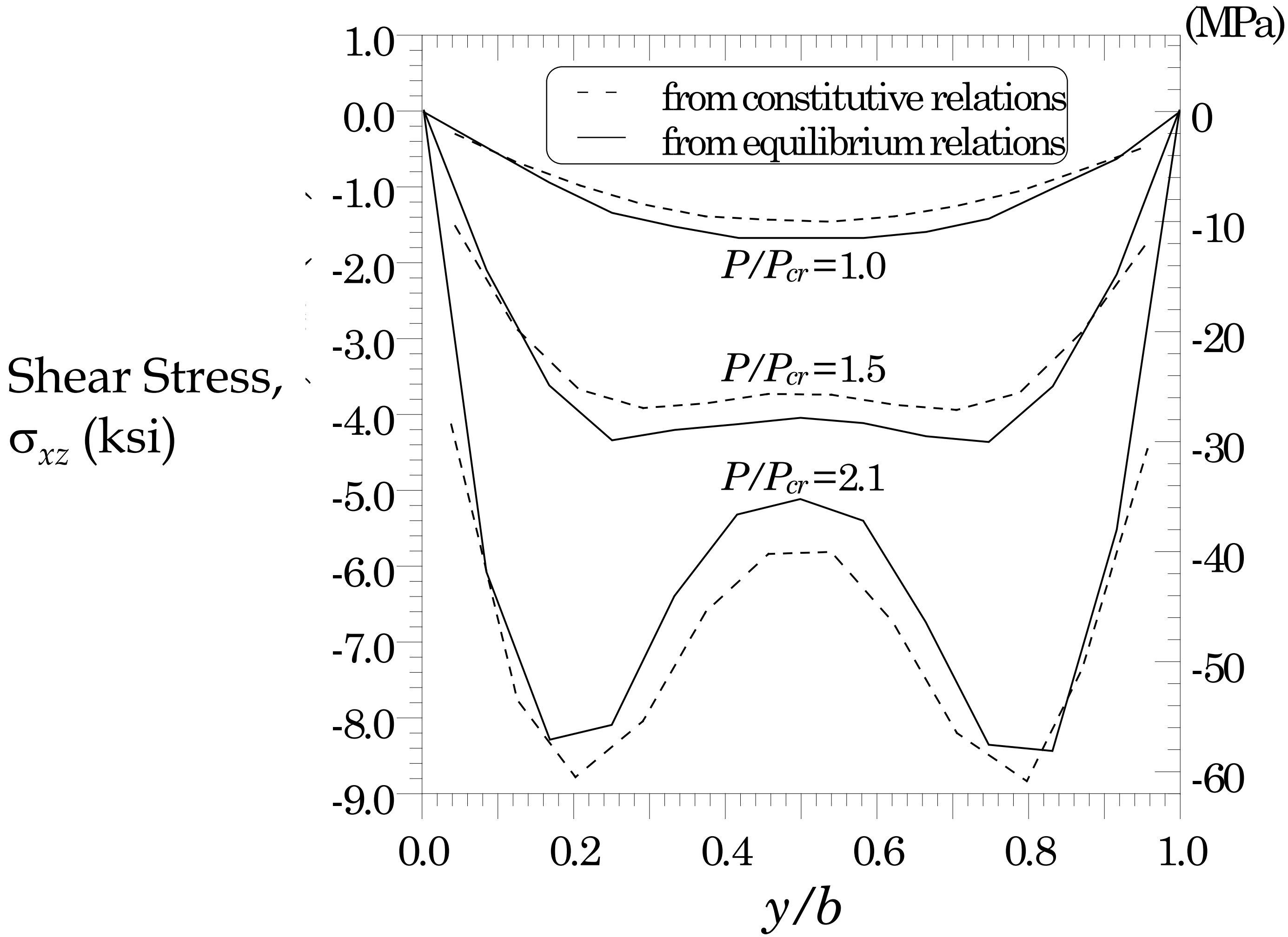




Postbuckling of a Composite Panel (Stress Distributions)



Postbuckling of a Composite Panel (Stress Distributions)



SUMMARY

- **Description of continuum shell element (principle of virtual displacements** **conjugate pairs, decompositions, approximation of the geometry and displacement field)**

- **Post-buckling analysis of composite panels and comparison with experimental results** **(problem description, boundary conditions, fringe patters of the displacement field, post-buckling path; failure analysis)**