FUNCTIONALLY GRADED BEAMS AND PLATES

- Euler-Bernoulli beam theory
- Timoshenko beam theory
- First-order shear deformation theory of plates

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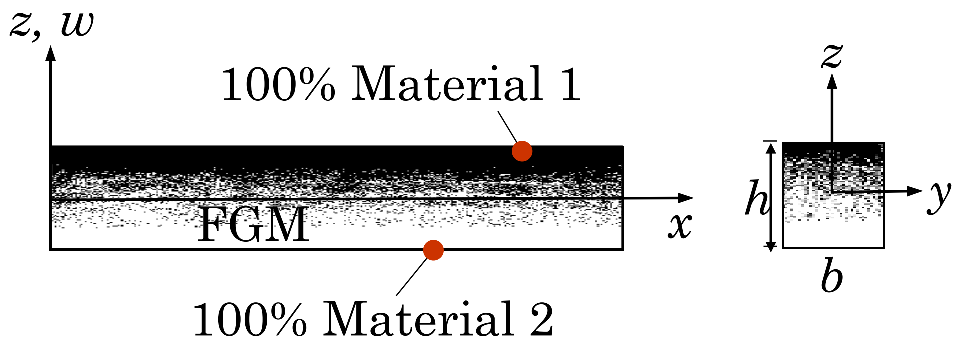
MATERIAL VARIATION THROUGH BEAM HEIGHT



$$P(z,T) = \left[P_1(T) - P_2(T)\right] f(z) + P_2(T), \quad f(z) = \left(\frac{1}{2} + \frac{z}{h}\right)^n$$

$$P_\alpha(T) = c_0\left(c_{-1}T^{-1} + 1 + c_1 T + c_2 T^2 + c_3 T^3\right), \; \alpha = c \text{ or } m$$

EULER-BERNOULLI BEAM THEORY

Equilibrium Equations (nonlinear)

Equilibrium equations

$$\delta u: \quad -\left(\frac{dN_{xx}}{dx} + f\right) = 0,$$

$$\delta w: \quad -\frac{d^2 M_{xx}}{dx^2} - \frac{d}{dx}\left(N_{xx}\frac{dw}{dx}\right) - q = 0$$

Boundary conditions

$$\text{Specify}: \quad u \quad \text{or} \quad N_{xx}$$

$$w \quad \text{or} \quad V_{xz} \equiv \left(N_{xx}\frac{dw}{dx} + \frac{dM_{xx}}{dx}\right)$$

$$\theta_x \equiv -\frac{dw}{dx} \quad \text{or} \quad M_{xx}$$

Euler-Bernoulli Beam Theory (EBT)

Constitutive Relations (nonlinear)

$$N_{xx} = \int_A \sigma_{xx}\, dA = \int_A E\left(\frac{du}{dx} + \frac{1}{2}\left(\frac{dw}{dx}\right)^2 - z\frac{d^2w}{dx^2}\right) dA$$

$$= A_{xx}\left[\frac{du}{dx} + \frac{1}{2}\left(\frac{dw}{dx}\right)^2\right] - B_{xx}\frac{d^2w}{dx^2}$$

$$M_{xx} = \int_A \sigma_{xx}\cdot z\, dA = \int_A E\left(\frac{du}{dx} + \frac{1}{2}\left(\frac{dw}{dx}\right)^2 - z\frac{d^2w}{dx^2}\right) z\, dA$$

$$= B_{xx}\left[\frac{du}{dx} + \frac{1}{2}\left(\frac{dw}{dx}\right)^2\right] - D_{xx}\frac{d^2w}{dx^2}$$

$$A_{xx} = \int_A E(z,T)\, dA,\ \ B_{xx} = \int_A zE(z,T)\, dA,\ \ D_{xx} = \int_A z^2E(z,T)\, dA$$

Equations of Equilibrium in Terms of Displacements (nonlinear case)

$$-\frac{d}{dx}\left(A_{xx}\left[\frac{du}{dx}+\frac{1}{2}\left(\frac{dw}{dx}\right)^2\right]-B_{xx}\frac{d^2w}{dx^2}\right)-f=0$$

$$\frac{d^2}{dx^2}\left\{B_{xx}\left[\frac{du}{dx}+\frac{1}{2}\left(\frac{dw}{dx}\right)^2\right]-D_{xx}\frac{d^2w}{dx^2}\right\}$$

$$-\frac{d}{dx}\left(\frac{dw}{dx}\left\{A_{xx}\left[\frac{du}{dx}+\frac{1}{2}\left(\frac{dw}{dx}\right)^2\right]-B_{xx}\frac{d^2w}{dx^2}\right\}\right)-q=0$$

TIMOSHENKO BEAM THEORY

Constitutive Relations (Nonlinear)

$$N_{xx} = \int_A \sigma_{xx}\, dA = \int_A E\left[\frac{du}{dx} + \frac{1}{2}\left(\frac{dw}{dx}\right)^2 + z\frac{d\phi_x}{dx}\right] dA$$

$$= A_{xx}\left[\frac{du}{dx} + \frac{1}{2}\left(\frac{dw}{dx}\right)^2\right] + B_{xx}\frac{d\phi_x}{dx}$$

$$M_{xx} = \int_A \sigma_{xx} z\, dA = \int_A E\left[\frac{du}{dx} + \frac{1}{2}\left(\frac{dw}{dx}\right)^2 + z\frac{d\phi_x}{dx}\right] z\, dA$$

$$= B_{xx}\left[\frac{du}{dx} + \frac{1}{2}\left(\frac{dw}{dx}\right)^2\right] + D_{xx}\frac{d\phi_x}{dx}$$

$$Q_{xz} = K_s \int_A \sigma_{xz}\, dA = \int_A GK_s\left(\phi_x + \frac{dw}{dx}\right) dA = S_{xz}\left(\phi_x + \frac{dw}{dx}\right)$$

TIMOSHENKO BEAM THEORY

Equations of Equilibrium (nonlinear)

Equilibrium equations

$$\delta u: \quad -\left(\frac{dN_{xx}}{dx} + f\right) = 0$$

$$\delta w: \quad -\frac{dQ_{xz}}{dx} - \frac{d}{dx}\left(N_{xx}\frac{dw}{dx}\right) - q = 0$$

$$\delta \phi_x: \quad -\frac{dM_{xx}}{dx} + Q_{xz} = 0$$

Boundary conditions

$$\text{Specify}: \quad u \quad \text{or} \quad N_{xx}$$

$$w \quad \text{or} \quad V_{xz} \equiv \left(N_{xx}\frac{dw}{dx} + Q_{xz}\right)$$

$$\phi_x \quad \text{or} \quad M_{xx}$$

TIMOSHENKO BEAM THEORY

Governing equations (nonlinear) in terms of the generalized displacements

$$-\frac{d}{dx}\left\{A_{xx}\left[\frac{du}{dx}+\frac{1}{2}\left(\frac{dw}{dx}\right)^2\right]+B_{xx}\frac{d\phi_x}{dx}\right\}-f=0$$

$$-\frac{d}{dx}\left[S_{xz}\left(\phi_x+\frac{dw}{dx}\right)\right]-\frac{d}{dx}\left[\frac{dw}{dx}\left\{A_{xx}\left[\frac{du}{dx}+\frac{1}{2}\left(\frac{dw}{dx}\right)^2\right]+B_{xx}\frac{d\phi_x}{dx}\right\}\right]-q=0$$

$$-\frac{d}{dx}\left\{B_{xx}\left[\frac{du}{dx}+\frac{1}{2}\left(\frac{dw}{dx}\right)^2\right]+D_{xx}\frac{d\phi_x}{dx}\right\}+S_{xz}\left(\phi_x+\frac{dw}{dx}\right)=0$$

FUNCTIONALLY GRADED PLATES

CONTENTS

- **Formulate a general thermo-mechanical model of through-the-thickness functionally graded plates using the first-order shear deformation plate theory, accounting for**

  (a) the von Karman nonlinearity

  (b) temperature dependent properties

- **Present numerical results for certain cases**

Kinematics of the FSDT

Displacement Field

$$u_1(x,y,z,t) = u(x,y,t) + z\phi_x(x,y,t)$$
$$u_2(x,y,z,t) = v(x,y,t) + z\phi_y(x,y,t)$$
$$u_3(x,y,z,t) = w(x,y,t)$$

Strain Field with the von Karman nonlinearity

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \gamma_{yz}^{(0)} \\ \gamma_{xz}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} + z \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{yz}^{(1)} \\ \gamma_{xz}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \\ \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \\ \frac{\partial w}{\partial y} + \phi_y \\ \frac{\partial w}{\partial x} + \phi_x \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \end{Bmatrix} + z \begin{Bmatrix} \frac{\partial \phi_x}{\partial x} \\ \frac{\partial \phi_y}{\partial y} \\ 0 \\ 0 \\ \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \end{Bmatrix}$$

Constitutive Relations

Inplane stresses

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} - \alpha \, \Delta T \\ \varepsilon_{yy} - \alpha \, \Delta T \\ \gamma_{xy} \end{Bmatrix}$$

$$Q_{11} = Q_{22} = \frac{E}{1-\nu^2}, \quad Q_{12} = \nu Q_{11}, \quad Q_{44} = Q_{55} = Q_{66} = \frac{E}{2(1+\nu)} = G$$

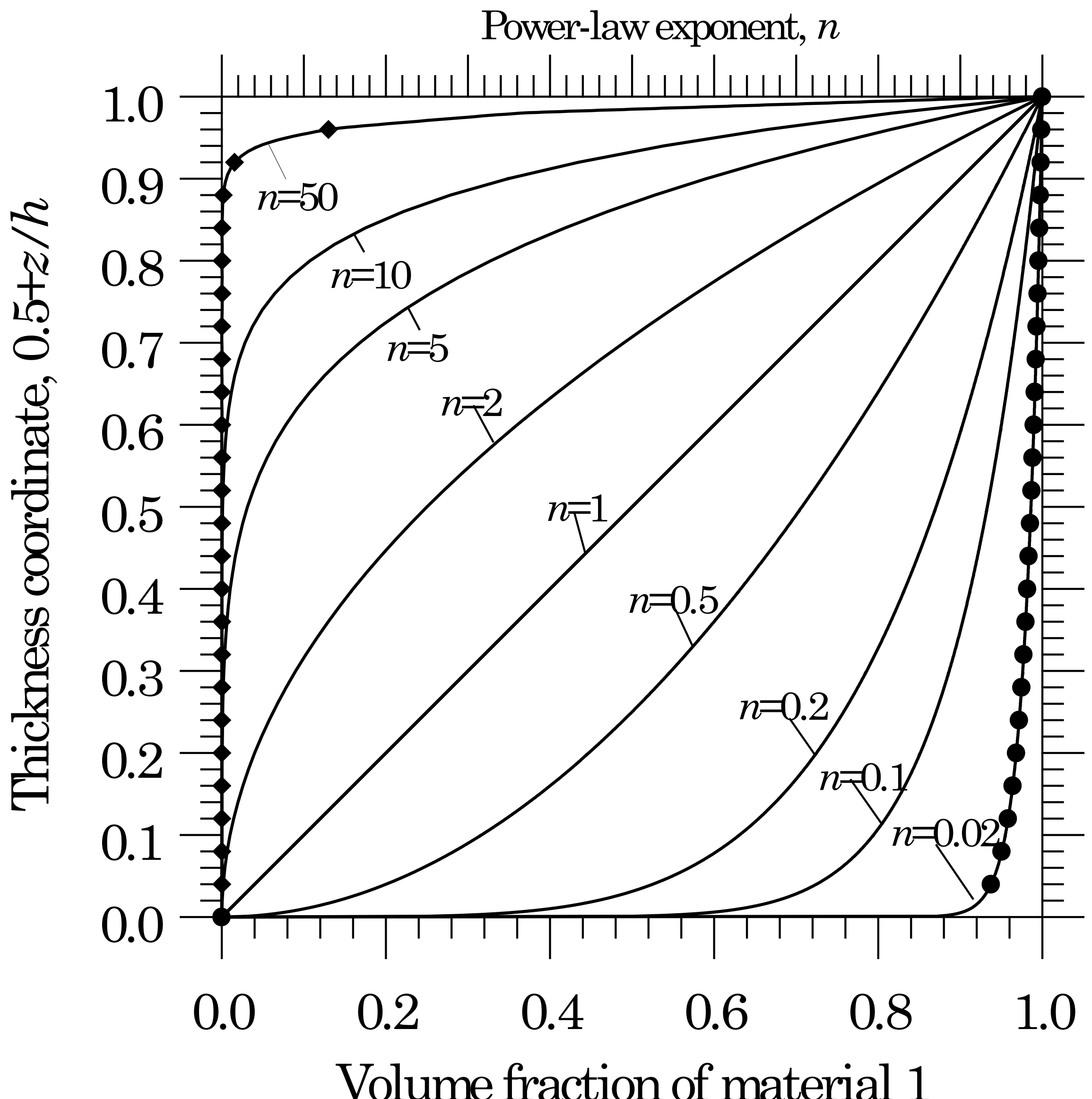
Transverse stresses

$$\sigma_{xz} = G\gamma_{xz}, \quad \sigma_{yz} = G\gamma_{yz}$$

Property variation

$$G = \frac{E}{2(1+\nu)}, \boldsymbol{E(z,T)} = [E_c(T) - E_m(T)] \left(\frac{1}{2} + \frac{z}{h}\right)^n + E_m(T)$$

Variation of metal volume fraction through the plate thickness



Equations of Motion

with the von Karaman Nonlinearity

$$\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} + f_x = I_0 \frac{\partial^2 u}{\partial t^2} + I_1 \frac{\partial^2 \phi_x}{\partial t^2}$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} + f_y = I_0 \frac{\partial^2 v}{\partial t^2} + I_1 \frac{\partial^2 \phi_y}{\partial t^2}$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + \frac{\partial}{\partial x}\left(N_{xx}\frac{\partial w}{\partial x} + N_{xy}\frac{\partial w}{\partial y}\right)$$

$$+ \frac{\partial}{\partial y}\left(N_{xy}\frac{\partial w}{\partial x} + N_{yy}\frac{\partial w}{\partial y}\right) + q = I_0 \frac{\partial^2 w}{\partial t^2}$$

$$\frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = I_2 \frac{\partial^2 \phi_x}{\partial t^2} + I_1 \frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_{yy}}{\partial y} - Q_y = I_2 \frac{\partial^2 \phi_y}{\partial t^2} + I_1 \frac{\partial^2 v}{\partial t^2}$$

Stress Resultants and Mass Inertias

Stress resultants

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} dz, \quad \begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} z \, dz$$

$$(Q_x, Q_y) = K \int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_{xz}, \sigma_{yz}) dz$$

Mass inertias

$$(I_0, I_1, I_2) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \left[(\rho_c - \rho_m) \left(\frac{1}{2} + \frac{z}{h} \right)^n + \rho_m \right] \left(1, z, z^2 \right) dz$$

$$I_0 = \rho_m h \frac{R+n}{1+n}, \quad R = \frac{\rho_m}{\rho_c} \quad \text{(density ratio)}$$

$$I_1 = \rho_m h^2 \frac{n(R-1)}{2(1+n)(2+n)}$$

$$I_2 = \frac{\rho_m h^3}{12} \left[\frac{(6+3n+3n^2)R + (8n+3n^2+n^3)}{6+11n+6n^2+n^3} \right]$$

FGM Plate Constitutive Relations

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{11} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{11} & 0 \\ 0 & 0 & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} - \begin{Bmatrix} N_{xx}^{T} \\ N_{yy}^{T} \\ N_{xy}^{T} \end{Bmatrix}$$

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{11} & 0 \\ 0 & 0 & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{11} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} - \begin{Bmatrix} M_{xx}^{T} \\ M_{yy}^{T} \\ M_{xy}^{T} \end{Bmatrix}$$

$$(Q_x, Q_y) = K \int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_{xz}, \sigma_{yz}) dz = \frac{K E_m h}{2(1+\nu)} \left(\frac{M+n}{1+n} \right) (\gamma_{xz}, \gamma_{yz}) \equiv (A_{55}\gamma_{xz}, A_{44}\gamma_{yz})$$

$$\{N^T\} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \{\beta\} \Delta T \; dz, \quad \{M^T\} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \{\beta\} \Delta T \; z \; dz$$

$$\{\beta\} = [Q]\{\alpha\} = \begin{Bmatrix} (Q_{11} + Q_{12})\,\alpha \\ (Q_{12} + Q_{22})\,\alpha \\ 0 \end{Bmatrix}$$

Temperature Dependent Properties

$$P(z,T) = \left[P_c(T) - P_m(T)\right] f(z) + P_m(T), \quad f(z) = \left(\frac{1}{2} + \frac{z}{h}\right)^n$$

$$P_\alpha(T) = c_0\left[c_{-1}T^{-1} + 1 + c_1 T + c_2 T^2 + c_3 T^3\right], \ \alpha = c \text{ or } m$$

Table 2.1: Material properties of Zirconia.

Property	c_0	c_{-1}	$c_1 \times 10^4$	$c_2 \times 10^8$	$c_3 \times 10^{10}$
ρ, Density (kg/m^3)	5,700	0	0	0	0
k, Conductivity (W/m K)	1.7	0	1.276	664.85	0
α, Coefficient of thermal expansion (K)	12.7657×10^{-6}	0	–14.9	0.0001	–0.06775
ν, Poisson's ratio	0.2882	0	1.13345	0	0
C_v, Specific heat (J/kg K)	487.34279	0	3.04908	–6.037232	0
E, Young's modulus (Pa)	244.26596×10^9	0	–13.707	121.393	–3.681378

Temperature Dependent Properties

Table 2.2: Material properties of Ti6AlV.

Property	c_0	c_{-1}	$c_1 \times 10^4$	$c_2 \times 10^8$	$c_3 \times 10^{10}$
ρ, Density (kg/m^3)	4,429	0	0	0	0
k, Conductivity (W/m K)	1.20947	0	139.375	0	0
α, Coefficient of thermal expansion (K)	7.57876×10^{-6}	0	6.5	31.467	0
ν, Poisson's ratio	0.28838235	0	1.12136	0	0
C_v, Specific heat (J/kg K)	625.29692	0	−4.2238757	71.786536	0
E, Young's modulus (Pa)	122.55676×10^9	0	−4.58635	0	−3.681378

FGM Plate Stiffnesses

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \left[\left(Q_{ij}^{(c)}(T) - Q_{ij}^{(m)}(T) \right) \left(\frac{1}{2} + \frac{z}{h} \right)^n + Q_{ij}^{(m)}(T) \right] \left(1, z, z^2\right) dz$$

$$A_{ij}(T) = Q_{ij}^{(m)} h \frac{M+n}{1+n}$$

$$B_{ij}(T) = Q_{ij}^{(m)} h^2 \frac{n(R-1)}{2(1+n)(2+n)}$$

$$D_{ij}(T) = Q_{ij}^{(m)} \frac{h^3}{12} \left[\frac{(6+3n+3n^2)M + (8n+3n^2+n^3)}{6+11n+6n^2+n^3} \right]$$

Weak Forms of the Equations of Motion

$$0=\int_{\Omega^e}\left(\frac{\partial\psi_i}{\partial x}N_{xx}+\frac{\partial\psi_i}{\partial y}N_{xy}+I_0\psi_i\frac{\partial^2 u}{\partial t^2}\right)dx\,dy-\oint_{\Gamma^e}N_{nn}\psi_i\,ds$$

$$0=\int_{\Omega^e}\left(\frac{\partial\psi_i}{\partial x}N_{xy}+\frac{\partial\psi_i}{\partial y}N_{yy}+I_0\psi_i\frac{\partial^2 v}{\partial t^2}\right)dx\,dy-\oint_{\Gamma^e}N_{ns}\psi_i\,ds$$

$$0=\int_{\Omega^e}\left[\frac{\partial\psi_i}{\partial x}\left(Q_x+\boxed{N_{xx}\frac{\partial w}{\partial x}+N_{xy}\frac{\partial w}{\partial y}}\right)+\frac{\partial\psi_i}{\partial y}\left(Q_y+\boxed{N_{xy}\frac{\partial w}{\partial x}+N_{yy}\frac{\partial w}{\partial y}}\right)\right.$$
$$\left.+I_0\psi_i\frac{\partial^2 w}{\partial t^2}-\psi_i q\right]dx\,dy-\oint_{\Gamma^e}Q_n\psi_i\,ds$$

$$0=\int_{\Omega^e}\left(\frac{\partial\psi_i}{\partial x}M_{xx}+\frac{\partial\psi_i}{\partial y}M_{xy}+\psi_iQ_x+I_2\psi_i\frac{\partial^2\phi_x}{\partial t^2}\right)dx\,dy-\oint_{\Gamma^e}M_{nn}\psi_i\,ds$$

$$0=\int_{\Omega^e}\left(\frac{\partial\psi_i}{\partial x}M_{xy}+\frac{\partial\psi_i}{\partial y}M_{yy}+\psi_iQ_y+I_2\psi_i\frac{\partial^2\phi_y}{\partial t^2}\right)dx\,dy-\oint_{\Gamma^e}M_{ns}\psi_i\,ds$$

Semidiscrete Finite Element Model

Approximations

$$u(x,y,t)=\sum_{j=1}^{n}\Delta_j^1(t)\psi_j(x,y),\quad v(x,y,t)=\sum_{j=1}^{n}\Delta_j^2(t)\psi_j(x,y)$$

$$w(x,y,t)=\sum_{j=1}^{n}\Delta_j^3(t)\psi_j(x,y)$$

$$\phi_x(x,y,t)=\sum_{j=1}^{n}\Delta_j^4(t)\psi_j(x,y),\quad \phi_y(x,y,t)=\sum_{j=1}^{n}\Delta_j^5(t)\psi_j(x,y)$$

Finite element equations

$$0=\sum_{\beta=1}^{5}\sum_{j=1}^{n}M_{ij}^{\alpha\beta}\ddot{\Delta}_j^{\beta}+\sum_{\beta=1}^{5}\sum_{j=1}^{n}K_{ij}^{\alpha\beta}\Delta_j^{\beta}-F_i^{\alpha}\equiv R_i^{\alpha}$$

Thermomechanical Coupling

Equilibrium equation of transverse forces

$$0 = \int_{\Omega^e} \left[\frac{\partial \psi_i}{\partial x} \left(Q_x + N_{xx} \frac{\partial w}{\partial x} + N_{xy} \frac{\partial w}{\partial y} \right) + \frac{\partial \psi_i}{\partial y} \left(Q_y + N_{xy} \frac{\partial w}{\partial x} + N_{yy} \frac{\partial w}{\partial y} \right) + I_0 \psi_i \frac{\partial^2 w}{\partial t^2} - \psi_i q \right] dx\, dy - \oint_{\Gamma^e} Q_n \psi_i \, ds$$

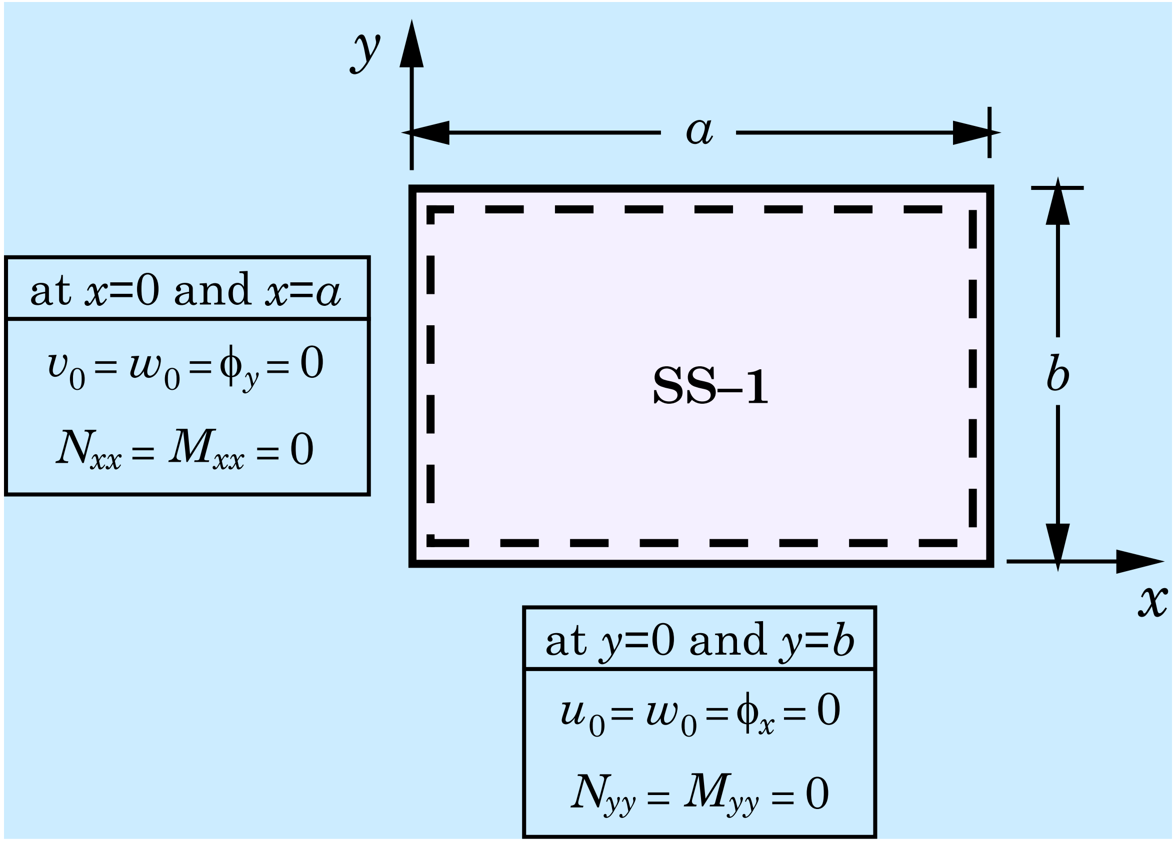
Contribution of the thermal forces to the force vector

$$\int_{\Omega^e} \left(\frac{\partial \psi_i}{\partial x} N_{xx}^T \frac{\partial w}{\partial x} + \frac{\partial \psi_i}{\partial y} N_{yy}^T \frac{\partial w}{\partial y} \right) dxdy$$

OR, contribution of the thermal forces to the stiffness matrix (making it stiffer)

$$\sum_{j=1}^{npe} w_j \int_{\Omega^e} \left(N_{xx}^T \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + N_{yy}^T \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} \right) dxdy$$

Boundary Conditions of a Simply Supported Plate



Aluminum and Zirconia Material Properties

Aluminum (bottom surface)

$$E_1 = 70 \text{ Gpa}, \ \nu = 0.3, \ \rho = 2707 \text{ kg/m}^3,$$

$$k = 204 \text{ W/(m.K)}, \ \alpha = 23 \times 10^{-6} \ / \ ^{o}\text{C}$$

Zirconia (top surface)

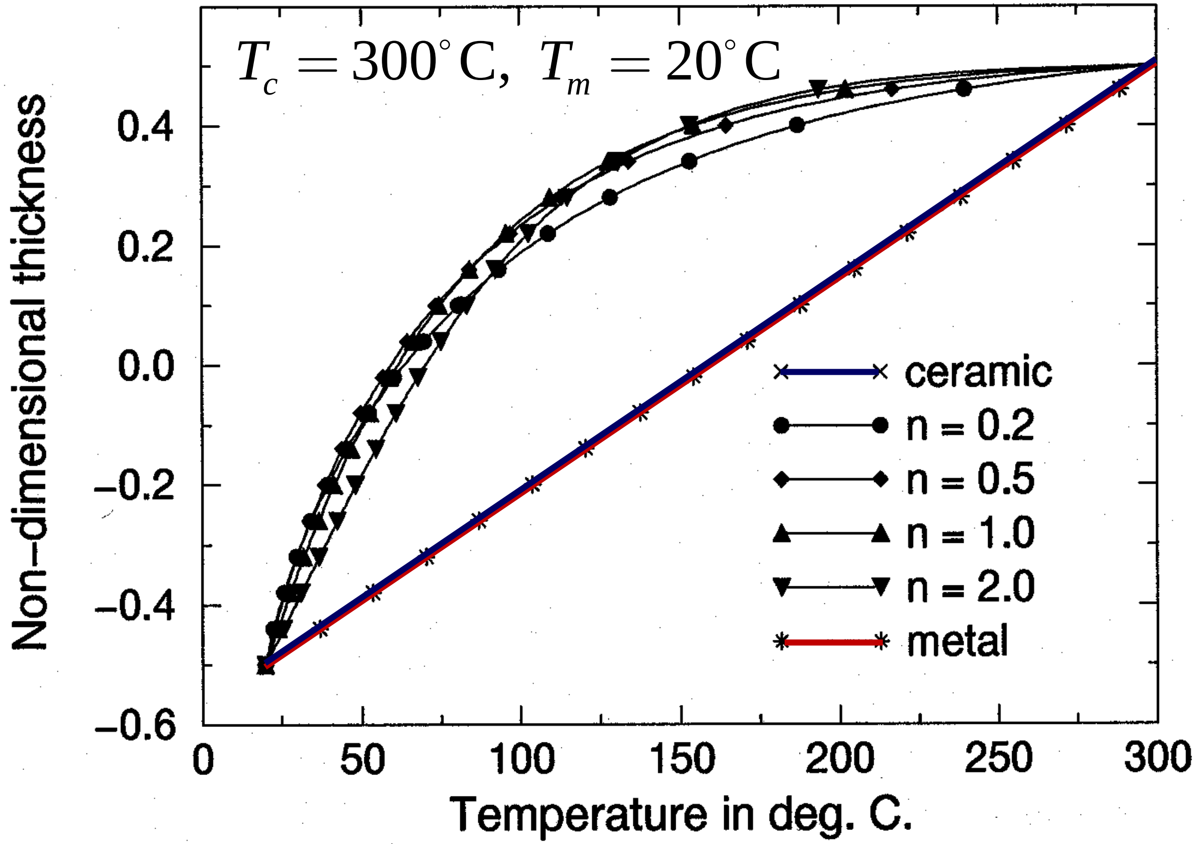
$$E_1 = 151 \text{ Gpa}, \ \nu = 0.3, \ \rho = 3000 \text{ kg/m}^3,$$

$$k = 2.09 \text{ W/(m.K)}, \ \alpha = 23 \times 10^{-5} \ / \ ^{o}\text{C}$$

Temperature variation through the thickness of the plate

$$-\frac{d}{dz}\left(k(z)\frac{dT}{dz}\right)=0$$

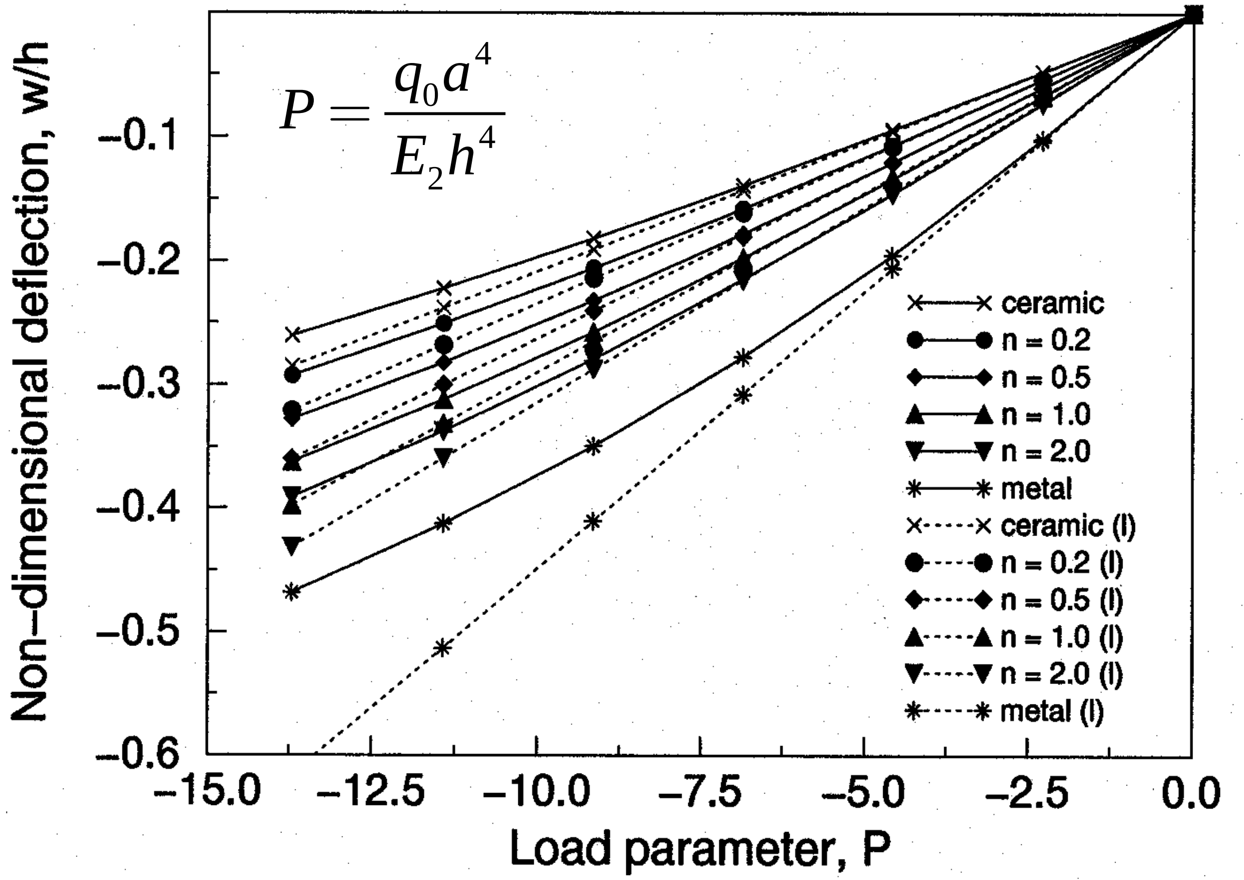
$$T=T_c \quad \text{at} \quad z=h/2; \quad T=T_m \quad \text{at} \quad z=-h/2$$



aluminum-zirconia FGM plate

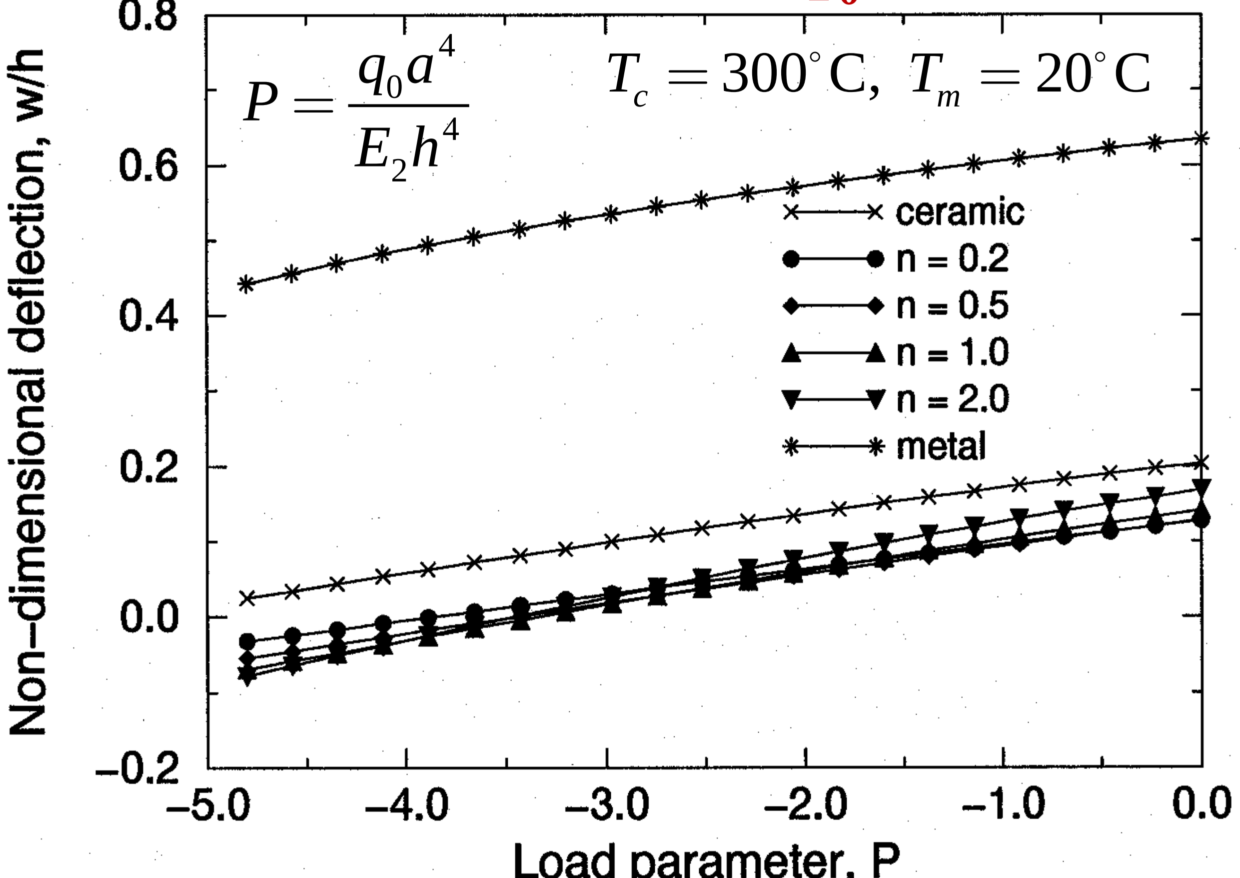
Center deflection vs load parameter for a simply supported FGM plate under uniform pressure

$a = 0.2\text{m}$, $h = 0.01\text{m}$, $q_0 = -10^4\ \text{N/m}^2$



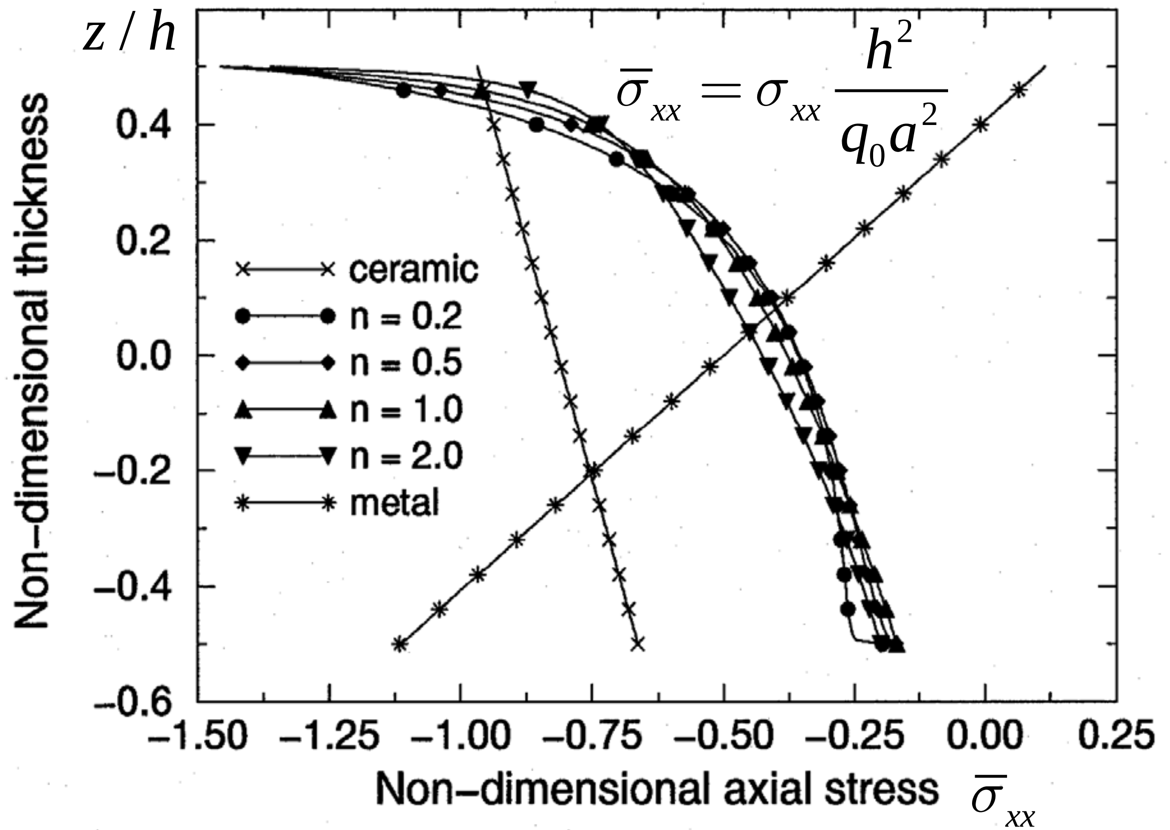
Center deflection vs load parameter for a simply supported FGM plate under uniform pressure and temperature variation

$a = 0.2\text{m}, h = 0.01\text{m}, q_0 = -10^4 \text{ N/m}^2$



Axial stress in a simply supported FGM plate under uniform loading and temperature distribution

$a = 0.2\text{m},\ h = 0.01\text{m},\ q_0 = -10^4\ \text{N/m}^2$



$T_c = 300^\circ\text{C},\ T_m = 20^\circ\text{C}$

SUMMARY, CONCLUSIONS AND FUTURE WORK

- A general thermomechanical models of functionally graded beams and plates are developed using the first-order shear deformation theories, accounting for the von Karman nonlinearity.
- Finite element models of the FGM plates are developed and numerical results are presented.
- The thermomechnical coupling in the nonlinear case makes the FGM plates to have a response that is NOT in between the ceramic and metal plates.
- Nonlinear analysis of FGM plates with temperature-dependent material properties is awaiting attention.