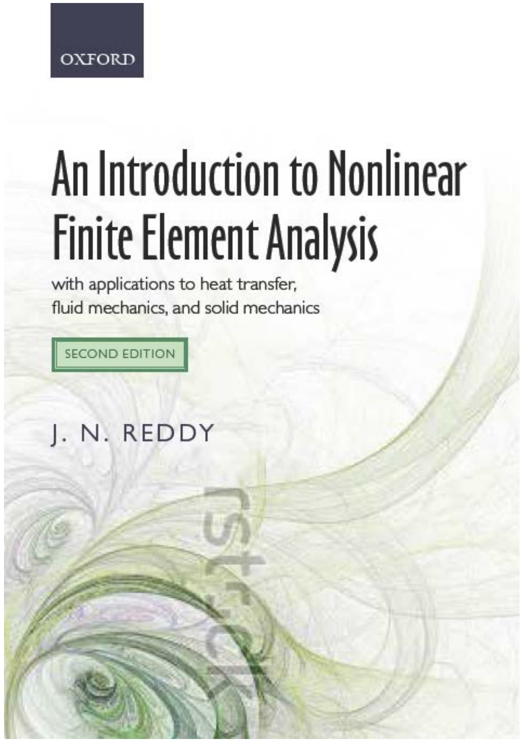
# MEEN 673:
# NONLINEAR FINITE ELEMENT ANALYSIS

## A REVIEW OF THE EQUATIONS OF MECHANICS

**Read: Chapter 2**




## CONTENTS

- Continuum assumption
- Kinematics of deformation
- Kinetics: Stress vector
- Cauchy's formula
- Balance of linear momentum
- Balance of angular momentum
- Conservation of Energy

In the study of deformation and motion of solid bodies, we make the simplifying assumption for convenience that the matter is distributed continuously, without any macroscopic gaps or empty spaces; that is, we disregard the molecular structure of matter.

**Continuum assumption**

$$\boxed{\rho(\mathbf{x},t) \equiv \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V}}$$

The study of motion and deformation of a continuum can be broadly classified into four basic categories:

(1) **Kinematics** (strain-displacement equations)
(2) **Kinetics** (balance of linear and angular momentum)
(3) **Thermodynamics** (first and second laws of thermodynamics)
(4) **Constitutive equations** (stress-strain relations)

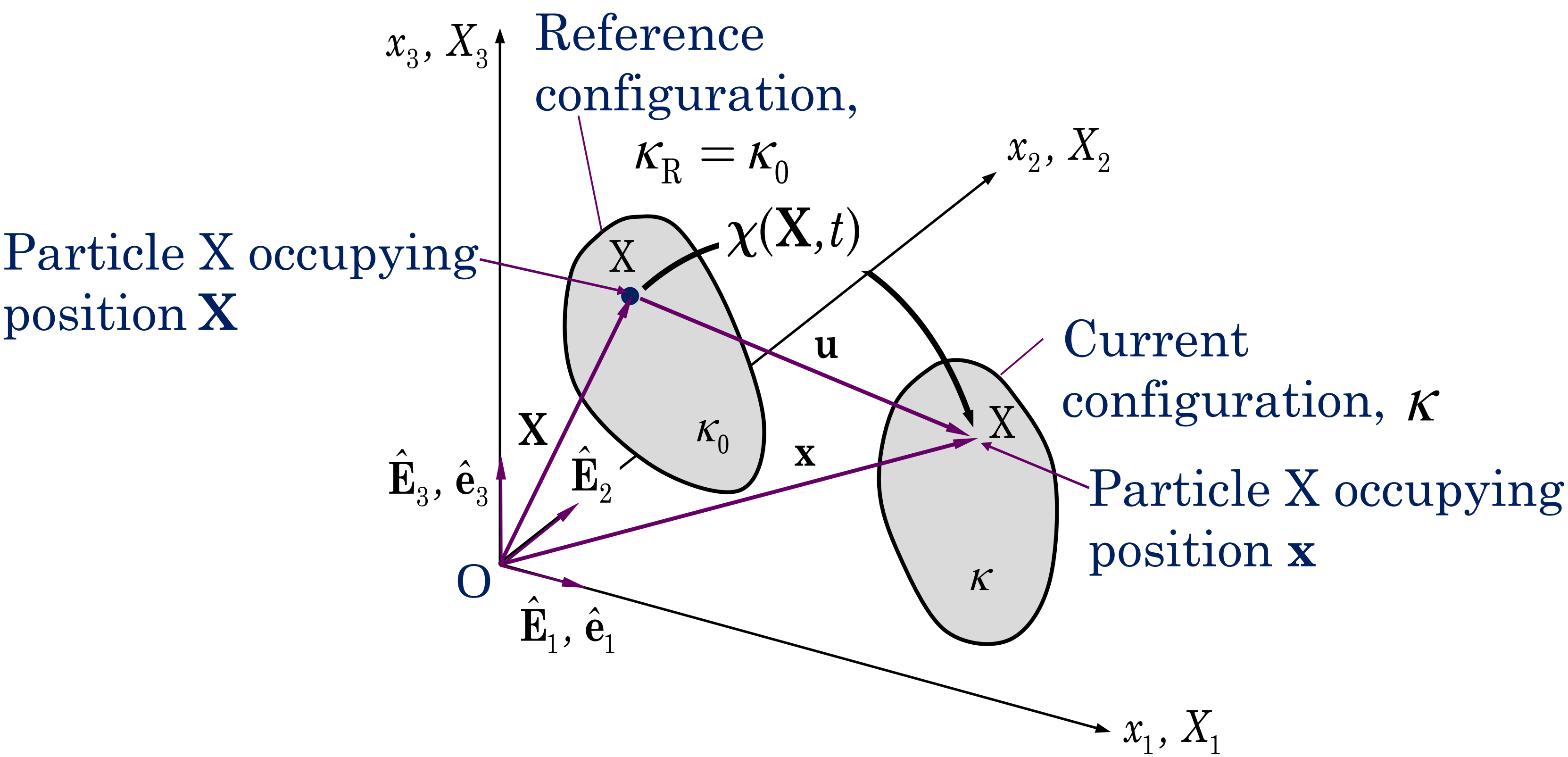
# KINEMATICS OF SOLIDS

**Material Description**: $\mathbf{x} = \chi(\mathbf{X},t), \quad \mathbf{X} = \chi(\mathbf{X},0)$

$(X_1, X_2, X_3)$ are the *material coordinates*.

$(x_1, x_2, x_3)$ are the *spatial* (current) *coordinates*.

Displacement vector: $\mathbf{u}(\mathbf{X},t) = \mathbf{x}(\mathbf{X},t) - \mathbf{X}$



# AN EXAMPLE

## Problem statement:

Consider the uniform deformation of a square block of side 2 units and initially centered at {\bf X}=(0,0). If the deformation is defined by the mapping

$$\boldsymbol{\chi}(\mathbf{X}) = \left(3.5 + X_1 + 0.5X_2\right)\hat{\mathbf{e}}_1 + \left(4 + X_2\right)\hat{\mathbf{e}}_2 + X_3\hat{\mathbf{e}}_3$$

(a) sketch the deformation, and (c) compute the displacements.

## Solution:

(a) From the given mapping, we have in matrix form, we have

$$x_1 = 3.5 + X_1 + 0.5X_2,\ x_2 = 4 + X_2,\ x_3 = X_3$$

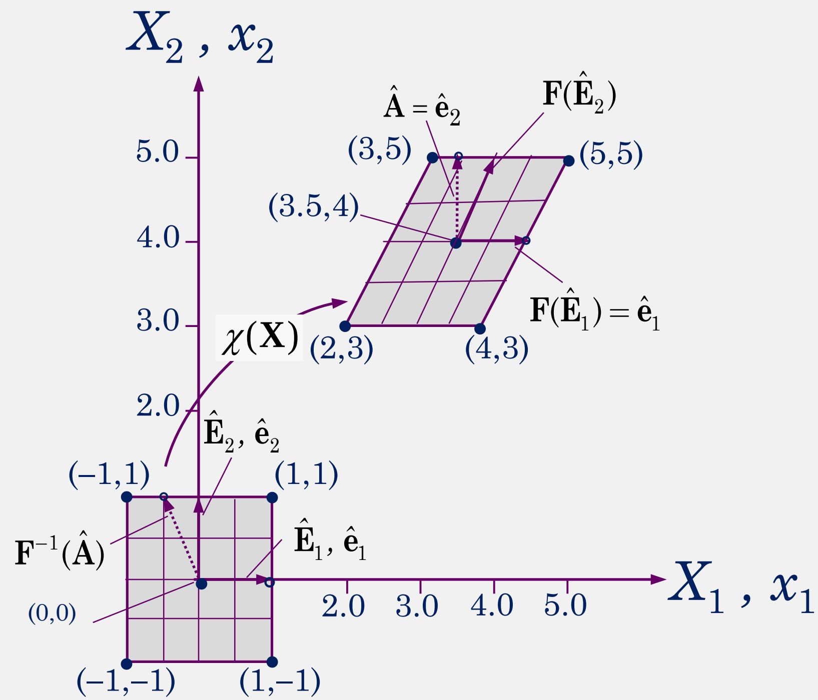
# AN EXAMPLE (continued)

(a) $x_1 = 3.5 + X_1 + 0.5X_2,$

$x_2 = 4 + X_2,\ \ x_3 = X_3.$

$X_1 = -1.5 + x_1 - 0.5x_2,$

$X_2 = -4 + x_2,\ X_3 = x_3.$




## Mapping

| $(X_1, X_2)$ | | $(x_1, x_2)$ |
|---|---|---|
| $(-1,-1)$ | $\rightarrow$ | $(2, 3)$ |
| $(1,\ -1)$ | $\rightarrow$ | $(4, 3)$ |
| $(1,\ 1)$ | $\rightarrow$ | $(5, 5)$ |
| $(-1,\ 1)$ | $\rightarrow$ | $(3,\ 5)$ |

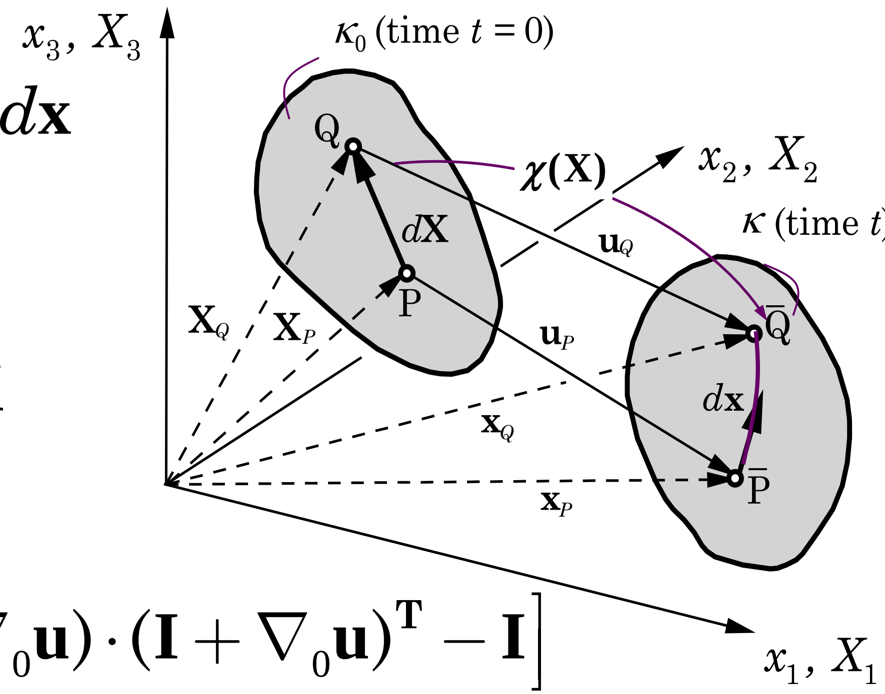
(b) $\mathbf{u} = \mathbf{x} - \mathbf{X} = \left(3.5 + 0.5X_2\right)\hat{\mathbf{e}}_1 + 4\,\hat{\mathbf{e}}_2 + 0\,\hat{\mathbf{e}}_3$

# GREEN-LAGRANGE STRAIN TENSOR

$$(dS)^2 = d\mathbf{X}\cdot d\mathbf{X},\ (ds)^2 = d\mathbf{x}\cdot d\mathbf{x}$$

Define the Green-Lagrange strain tensor **E** as

$$(ds)^2 - (dS)^2 \equiv 2d\mathbf{X}\cdot\mathbf{E}\cdot d\mathbf{X}$$




where **E** is

$$\mathbf{E} = \tfrac{1}{2}\left(\frac{d\mathbf{x}}{d\mathbf{X}}\cdot\frac{d\mathbf{x}}{d\mathbf{X}} - \mathbf{I}\right) = \tfrac{1}{2}\left[(\mathbf{I}+\nabla_0\mathbf{u})\cdot(\mathbf{I}+\nabla_0\mathbf{u})^{\mathbf{T}} - \mathbf{I}\right]$$

$$= \tfrac{1}{2}\left[\nabla_0\mathbf{u} + (\nabla_0\mathbf{u})^{\mathbf{T}} + (\nabla_0\mathbf{u})\cdot(\nabla_0\mathbf{u})^{\mathbf{T}}\right]$$

The rectangular Cartesian component form of **E** is

$$E_{IJ} = \frac{1}{2}\left(\frac{\partial u_I}{\partial X_J} + \frac{\partial u_J}{\partial X_I} + \frac{\partial u_K}{\partial X_I}\frac{\partial u_K}{\partial X_J}\right)$$

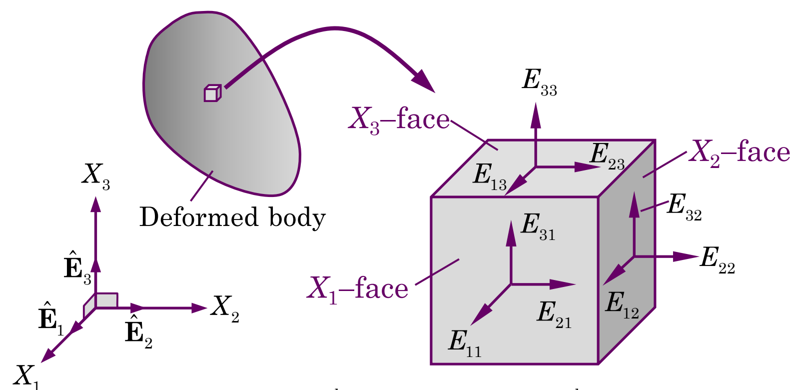
# GREEN-LAGRANGE STRAIN TENSOR

The rectangular Cartesian components in explicit form are given by

$$E_{11} = \frac{\partial u_1}{\partial X_1} + \frac{1}{2}\left[\left(\frac{\partial u_1}{\partial X_1}\right)^2 + \left(\frac{\partial u_2}{\partial X_1}\right)^2 + \left(\frac{\partial u_3}{\partial X_1}\right)^2\right],$$

$$E_{22} = \frac{\partial u_2}{\partial X_2} + \frac{1}{2}\left[\left(\frac{\partial u_1}{\partial X_2}\right)^2 + \left(\frac{\partial u_2}{\partial X_2}\right)^2 + \left(\frac{\partial u_3}{\partial X_2}\right)^2\right]$$

$$E_{33} = \frac{\partial u_3}{\partial X_3} + \frac{1}{2}\left[\left(\frac{\partial u_1}{\partial X_3}\right)^2 + \left(\frac{\partial u_2}{\partial X_3}\right)^2 + \left(\frac{\partial u_3}{\partial X_3}\right)^2\right]$$




$E_{IJ}$ – Strain in the $I^{th}$ direction on the $J^{th}$ face

$$E_{12} = \frac{1}{2}\left(\frac{\partial u_1}{\partial X_2} + \frac{\partial u_2}{\partial X_1} + \frac{\partial u_1}{\partial X_1}\frac{\partial u_1}{\partial X_2} + \frac{\partial u_2}{\partial X_1}\frac{\partial u_2}{\partial X_2} + \frac{\partial u_3}{\partial X_1}\frac{\partial u_3}{\partial X_2}\right)$$

$$E_{13} = \frac{1}{2}\left(\frac{\partial u_1}{\partial X_3} + \frac{\partial u_3}{\partial X_1} + \frac{\partial u_1}{\partial X_1}\frac{\partial u_1}{\partial X_3} + \frac{\partial u_2}{\partial X_1}\frac{\partial u_2}{\partial X_3} + \frac{\partial u_3}{\partial X_1}\frac{\partial u_3}{\partial X_3}\right)$$

$$E_{23} = \frac{1}{2}\left(\frac{\partial u_2}{\partial X_3} + \frac{\partial u_3}{\partial X_2} + \frac{\partial u_1}{\partial X_2}\frac{\partial u_1}{\partial X_3} + \frac{\partial u_2}{\partial X_2}\frac{\partial u_2}{\partial X_3} + \frac{\partial u_3}{\partial X_2}\frac{\partial u_3}{\partial X_3}\right)$$

# INFITESIMAL STRAIN TENSOR

If $\mathbf{E}$ is of the order $\mathrm{O}(\epsilon)$ in $\nabla_0\mathbf{u}$, then we mean

$$\frac{\partial u_I}{\partial X_J} = \mathrm{O}(\epsilon) \quad \text{as} \quad \epsilon \to 0$$

If terms of the order $\mathrm{O}(\epsilon^2)$ can be omitted, then

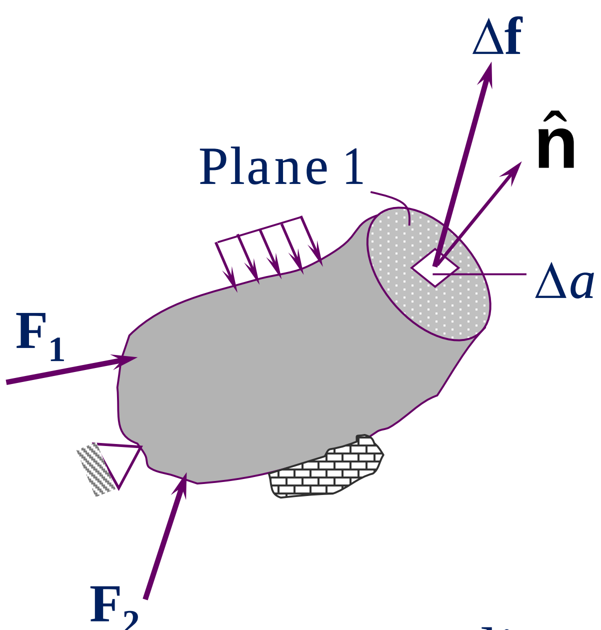
$$E_{IJ} = \frac{1}{2}\left(\frac{\partial u_I}{\partial X_J} + \frac{\partial u_J}{\partial X_I} + \frac{\partial u_K}{\partial X_I}\frac{\partial u_K}{\partial X_J}\right)$$

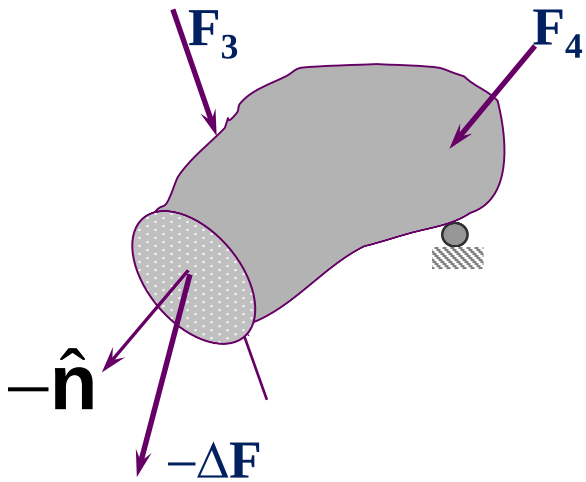
can be approximated as

$$E_{IJ} \approx \frac{1}{2}\left(\frac{\partial u_I}{\partial X_J} + \frac{\partial u_J}{\partial X_I}\right) = \mathrm{O}(\epsilon) \text{ as } \epsilon^2 \to 0$$

$$\mathbf{E} \approx \boldsymbol{\varepsilon} = \frac{1}{2}\left[\nabla_0\mathbf{u} + (\nabla_0\mathbf{u})^{\mathrm{T}}\right],$$ the infinitesimal strain tensor

# KINETICS: Stress Vector



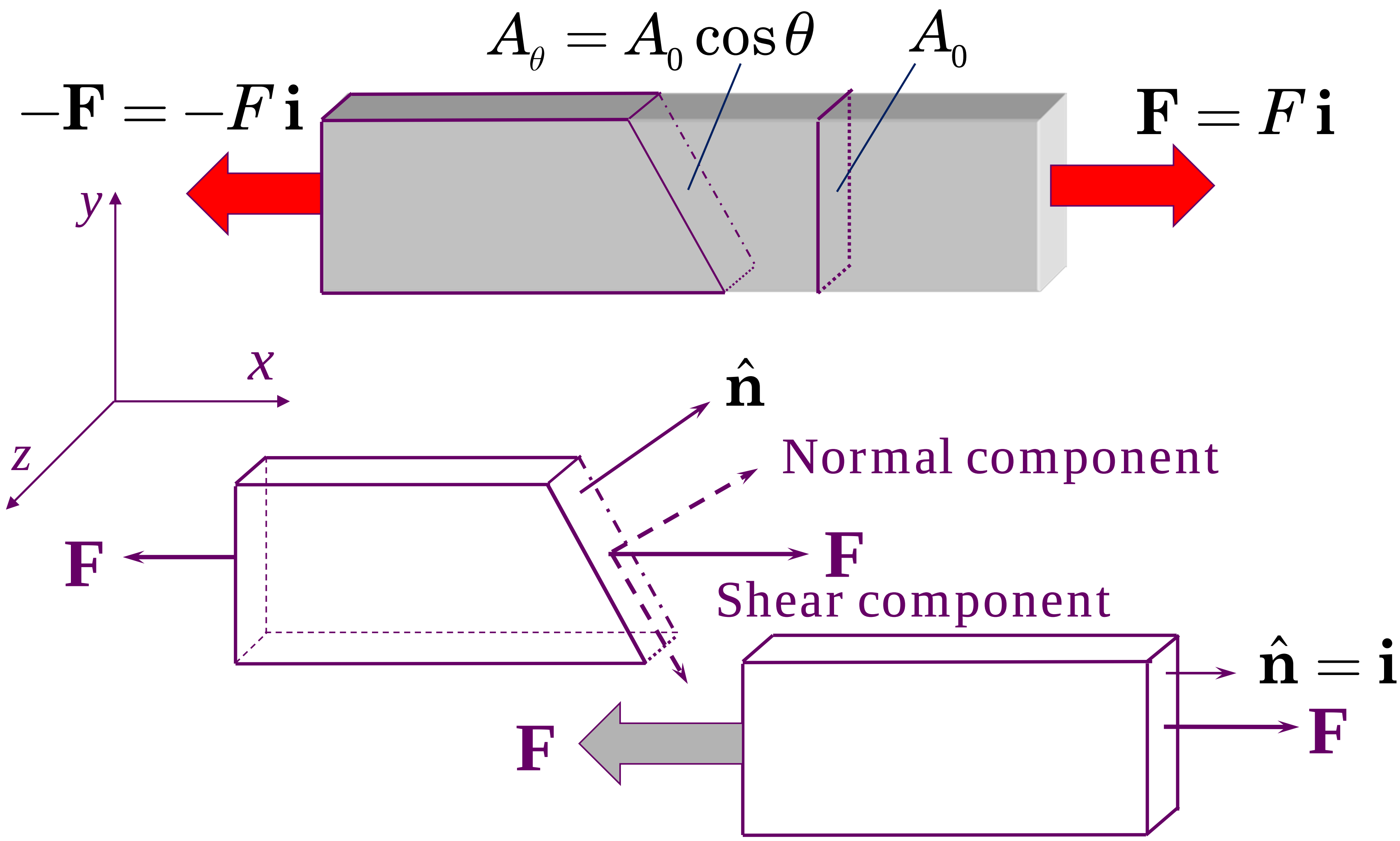




$$\mathbf{t}^{(\hat{\mathbf{n}})} = \lim_{\Delta a \to 0} \frac{\Delta \mathbf{f}}{\Delta a} \qquad \mathbf{t}^{(-\hat{\mathbf{n}})} = \lim_{\Delta a \to 0} \frac{-\Delta \mathbf{f}}{\Delta a}$$

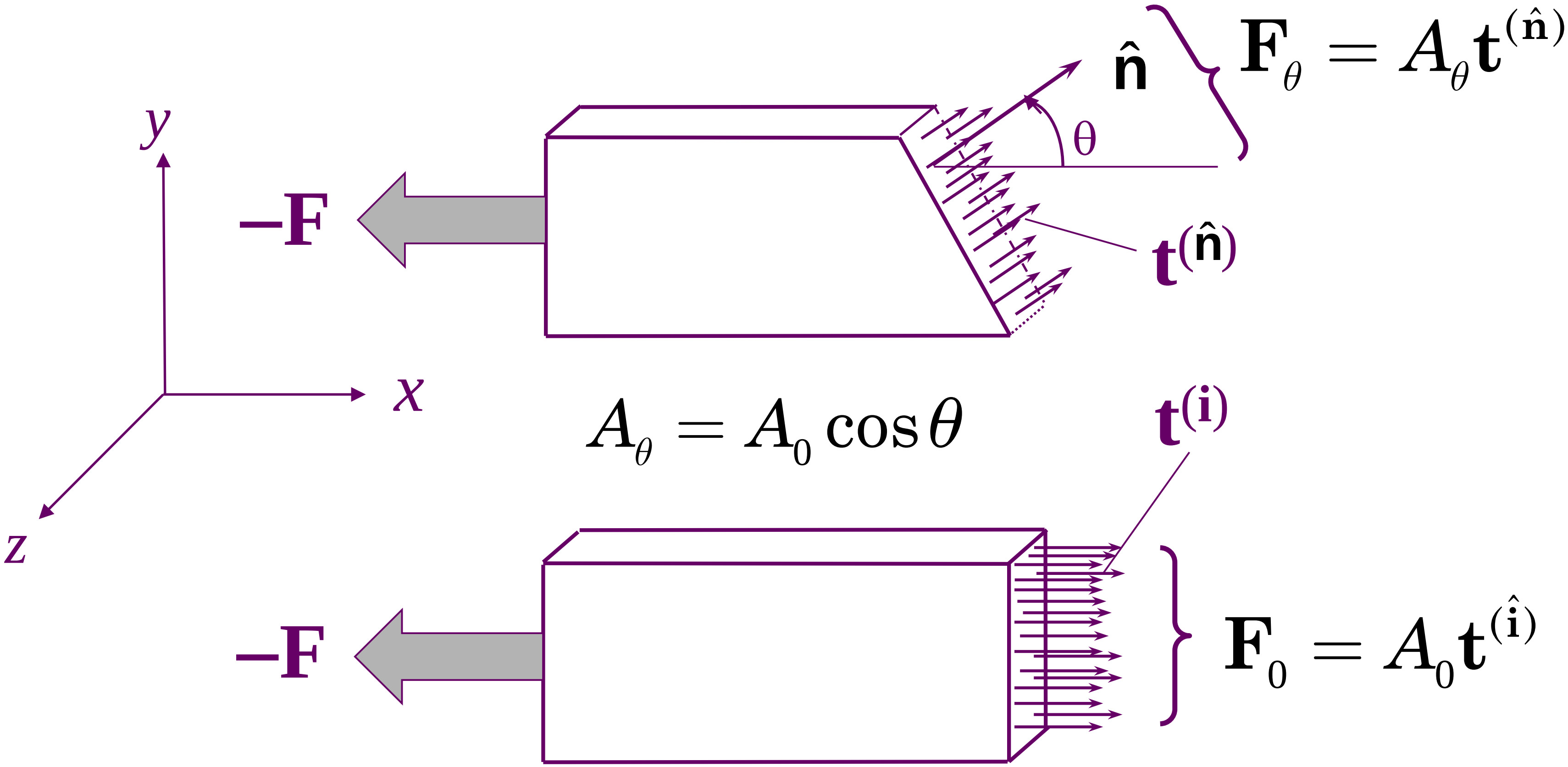
$$\boxed{\mathbf{t}^{(\hat{\mathbf{n}})} = -\mathbf{t}^{(-\hat{\mathbf{n}})}} \quad \text{or} \quad \boxed{\mathbf{t}^{(-\hat{\mathbf{n}})} = -\mathbf{t}^{(\hat{\mathbf{n}})}}$$

**Cauchy's Lemma**

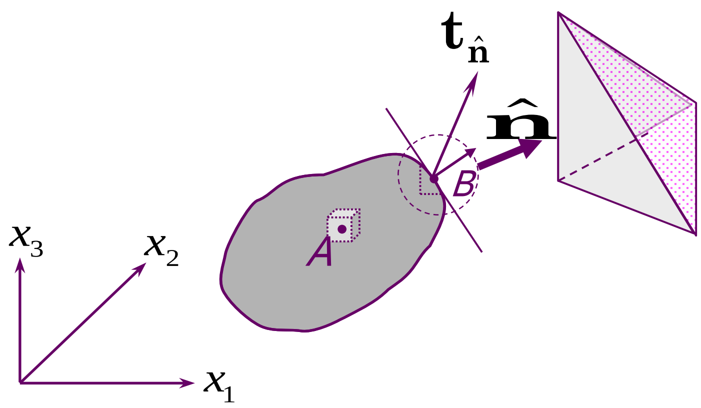
# Traction Vector: Examples

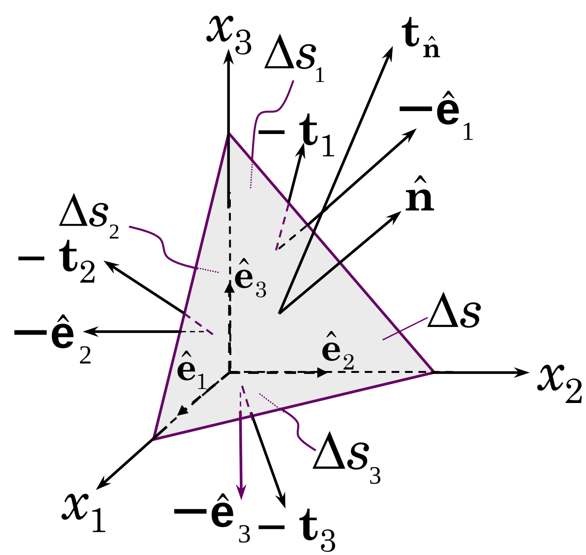


# Traction Vector: Examples



# CAUCHY STRESS TENSOR-1




$$\mathbf{t}\Delta s-\mathbf{t}_1\Delta s_1-\mathbf{t}_2\Delta s_2-\mathbf{t}_3\Delta s_3+\rho\Delta v\mathbf{f}=\rho\Delta v\mathbf{a}$$

$$\mathbf{t}=\mathbf{t}_1(\hat{\mathbf{e}}_1\cdot\hat{\mathbf{n}})+\mathbf{t}_2(\hat{\mathbf{e}}_2\cdot\hat{\mathbf{n}})+\mathbf{t}_3(\hat{\mathbf{e}}_3\cdot\hat{\mathbf{n}})+\rho\frac{\Delta h}{3}(\mathbf{a}-\mathbf{f})$$

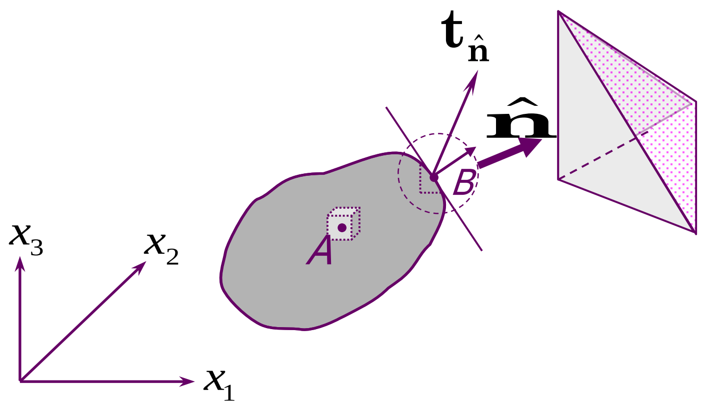
As $\Delta h\rightarrow 0$, we obtain

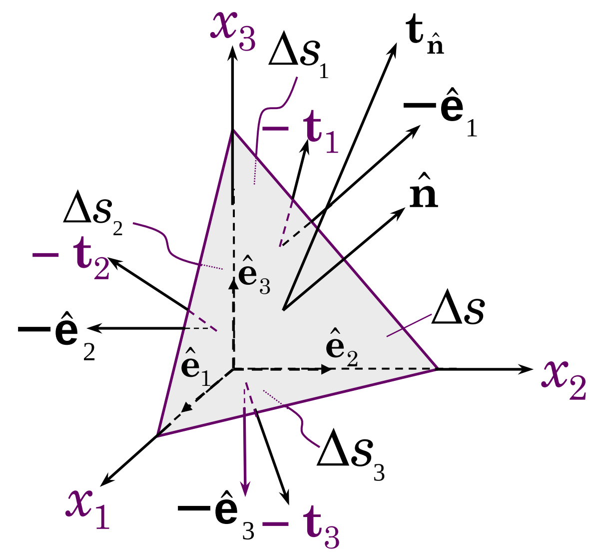
$$\mathbf{t}=\mathbf{t}_1(\hat{\mathbf{e}}_1\cdot\hat{\mathbf{n}})+\mathbf{t}_2(\hat{\mathbf{e}}_2\cdot\hat{\mathbf{n}})+\mathbf{t}_3(\hat{\mathbf{e}}_3\cdot\hat{\mathbf{n}})=\mathbf{t}_j\hat{\mathbf{e}}_j\cdot\hat{\mathbf{n}}\equiv\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}$$

$$\mathbf{t}^{(\hat{\mathbf{n}})}=\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}\quad\text{and}\quad\boldsymbol{\sigma}=\mathbf{t}_j\hat{\mathbf{e}}_j\quad\left[\; t_i^{(\hat{\mathbf{n}})}=\sigma_{ij}n_j;\;\; \mathbf{t}_i=\sigma_{ji}\,\hat{\mathbf{e}}_j,\;\; \boldsymbol{\sigma}=\sigma_{ij}\hat{\mathbf{e}}_i\hat{\mathbf{e}}_j\;\right]$$

plane direction

# CAUCHY'S FORMULA




$$\mathbf{t}\Delta s - \mathbf{t}_1\Delta s_1 - \mathbf{t}_2\Delta s_2 - \mathbf{t}_3\Delta s_3 + \rho\Delta v\mathbf{f} = \rho\Delta v\mathbf{a}$$

$$\mathbf{t} = (\hat{\mathbf{n}}\cdot\hat{\mathbf{e}}_1)\mathbf{t}_1 + (\hat{\mathbf{n}}\cdot\hat{\mathbf{e}}_2)\mathbf{t}_2 + (\hat{\mathbf{n}}\cdot\hat{\mathbf{e}}_3)\mathbf{t}_3 + \rho\frac{\Delta h}{3}(\mathbf{a} - \mathbf{f})$$

As $\Delta h \rightarrow 0$, we obtain

$$\mathbf{t} = (\hat{\mathbf{n}}\cdot\hat{\mathbf{e}}_1)\mathbf{t}_1 + (\hat{\mathbf{n}}\cdot\hat{\mathbf{e}}_2)\mathbf{t}_2 + (\hat{\mathbf{n}}\cdot\hat{\mathbf{e}}_3)\mathbf{t}_3 = (\hat{\mathbf{n}}\cdot\hat{\mathbf{e}}_i)\mathbf{t}_i \equiv \hat{\mathbf{n}}\cdot\boldsymbol{\sigma}$$

$$\mathbf{t}^{(\hat{\mathbf{n}})} = \hat{\mathbf{n}}\cdot\boldsymbol{\sigma} \quad \text{and} \quad \boldsymbol{\sigma} = \hat{\mathbf{e}}_i\mathbf{t}_i \quad \left[ t_i^{(\hat{\mathbf{n}})} = n_j\sigma_{ji}, \; \underset{\text{plane}}{\mathbf{t}_i} = \sigma_{ij}\underset{\text{direction}}{\hat{\mathbf{e}}_j} \right]$$

# AN EXAMPLE

**Problem**: Given the following stress tensor components in Cartesian coordinates

$$[\sigma] = \begin{bmatrix} 5 & -2 & 0 \\ -2 & -1 & 0 \\ 0 & 0 & -3 \end{bmatrix} (\text{MPa})$$

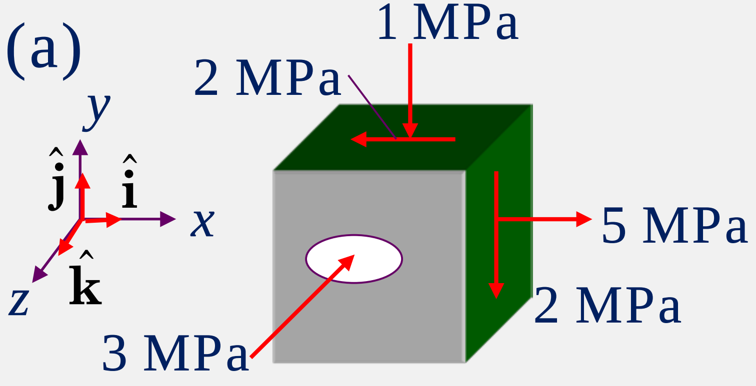
(a) Show the stress components on the stress cube.

(b) Determine the traction vectors $\mathbf{t}^{(\hat{\mathbf{i}})}$, $\mathbf{t}^{(\hat{\mathbf{j}})}$, and $\mathbf{t}^{(\hat{\mathbf{k}})}$

(c) Sketch the traction vectors on the stress cube.

**Solution**: We have

(a)



Stress cube

(b)

$$\mathbf{t}^{(\hat{\mathbf{i}})} = 5\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$$

$$\mathbf{t}^{(\hat{\mathbf{j}})} = -2\hat{\mathbf{i}} - \hat{\mathbf{j}}$$

$$\mathbf{t}^{(\hat{\mathbf{k}})} = -3\hat{\mathbf{k}}$$

# Example (continued)

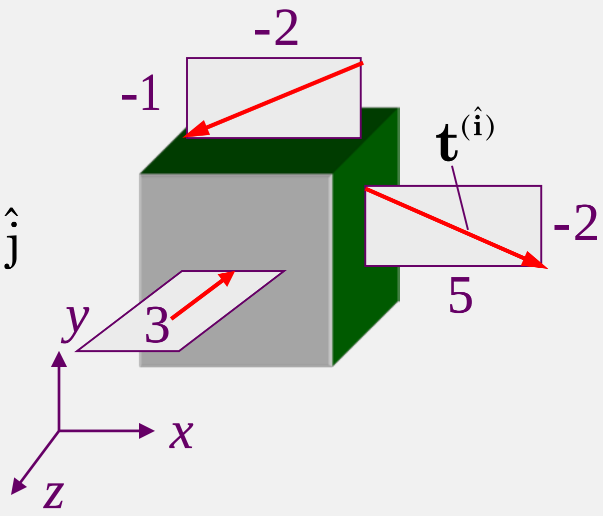
(b) Solution by use of Cauchy's formula

$$[\sigma]=\begin{bmatrix}5 & -2 & 0\\ -2 & -1 & 0\\ 0 & 0 & -3\end{bmatrix}(\text{MPa}),\qquad \begin{Bmatrix}t_x\\ t_y\\ t_z\end{Bmatrix}=\begin{bmatrix}5 & -2 & 0\\ -2 & -1 & 0\\ 0 & 0 & -3\end{bmatrix}\begin{Bmatrix}n_x\\ n_y\\ n_z\end{Bmatrix}$$

(c)

(i) When $\hat{\mathbf{n}}=\hat{\mathbf{i}}\ \ (n_x=1,\ n_y=0,\ n_z=0)$

$$\begin{Bmatrix}t_x\\ t_y\\ t_z\end{Bmatrix}^{(\hat{\mathrm{i}})}=\begin{bmatrix}5 & -2 & 0\\ -2 & -1 & 0\\ 0 & 0 & -3\end{bmatrix}\begin{Bmatrix}1\\ 0\\ 0\end{Bmatrix}=\begin{Bmatrix}5\\ -2\\ 0\end{Bmatrix}\Rightarrow \mathrm{t}^{(\hat{\mathrm{i}})}=5\hat{\mathrm{i}}-2\hat{\mathrm{j}}$$




(ii) When $\hat{\mathbf{n}}=\hat{\mathbf{j}}\ \ (n_x=0,\ n_y=1,\ n_z=0)$

$$\begin{Bmatrix}t_x\\ t_y\\ t_z\end{Bmatrix}^{(\hat{\mathrm{j}})}=\begin{bmatrix}5 & -2 & 0\\ -2 & -1 & 0\\ 0 & 0 & -3\end{bmatrix}\begin{Bmatrix}0\\ 1\\ 0\end{Bmatrix}=\begin{Bmatrix}-2\\ -1\\ 0\end{Bmatrix}\Rightarrow \mathrm{t}^{(\hat{\mathrm{j}})}=-2\hat{\mathrm{i}}-\hat{\mathrm{j}}$$

(iii) When $\hat{\mathbf{n}}=\hat{\mathbf{k}}\ \ (n_x=0,\ n_y=0,\ n_z=1)\Rightarrow \mathbf{t}^{(\hat{\mathbf{k}})}=-3\hat{\mathbf{k}}$

# AN EXAMPLE

**Problem statement**:

With reference to a rectangular Cartesian system $(x_1, x_2, x_3)$, the components of the stress dyadic at a certain point of a continuous medium are given by

$$[\boldsymbol{\sigma}] = \begin{bmatrix} 200 & 400 & 300 \\ 400 & 0 & 0 \\ 300 & 0 & -100 \end{bmatrix} \text{ psi.}$$

Determine stress vector and its normal and tangential components at the point on the plane

$$\phi(x_1, x_2, x_3) \equiv x_1 + 2x_2 + 2x_3 = \text{constant}$$

which is planning through the point.

# AN EXAMPLE

**Solution**:

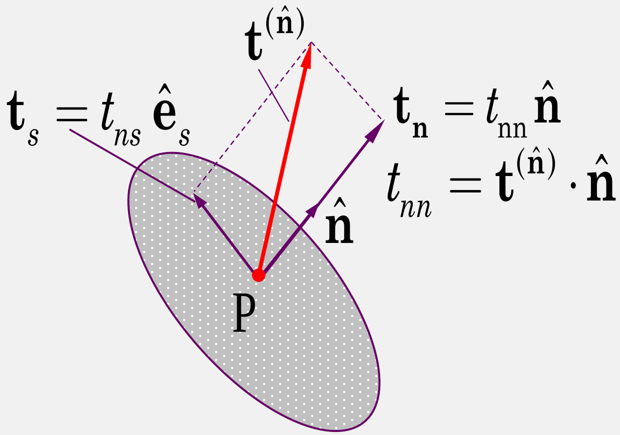
First, we should find the unit normal to the plane on which we are required to find the stress vector. The unit normal to the plane is

$$\hat{\mathbf{n}} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{1}{3}(\hat{\mathbf{e}}_1 + 2\hat{\mathbf{e}}_2 + 2\hat{\mathbf{e}}_3)$$

The components of the stress vector are

$$\begin{Bmatrix} t_1 \\ t_2 \\ t_3 \end{Bmatrix} = \begin{bmatrix} 200 & 400 & 300 \\ 400 & 0 & 0 \\ 300 & 0 & -100 \end{bmatrix} \frac{1}{3} \begin{Bmatrix} 1 \\ 2 \\ 2 \end{Bmatrix} = \frac{1}{3} \begin{Bmatrix} 1600 \\ 400 \\ 100 \end{Bmatrix} \text{ psi}$$

## Solution (continued):




The traction vector normal to the plane is given by

$$\mathbf{t}_{nn} = (\mathbf{t}(\hat{\mathbf{n}}) \cdot \hat{\mathbf{n}})\hat{\mathbf{n}} = \frac{2600}{9}\hat{\mathbf{n}}$$

$$= \frac{2600}{27}(\hat{\mathbf{e}}_1 + 2\hat{\mathbf{e}}_2 + 2\hat{\mathbf{e}}_3) \text{ psi}$$

and the traction vector projected onto the plane (i.e., shear traction) is given by

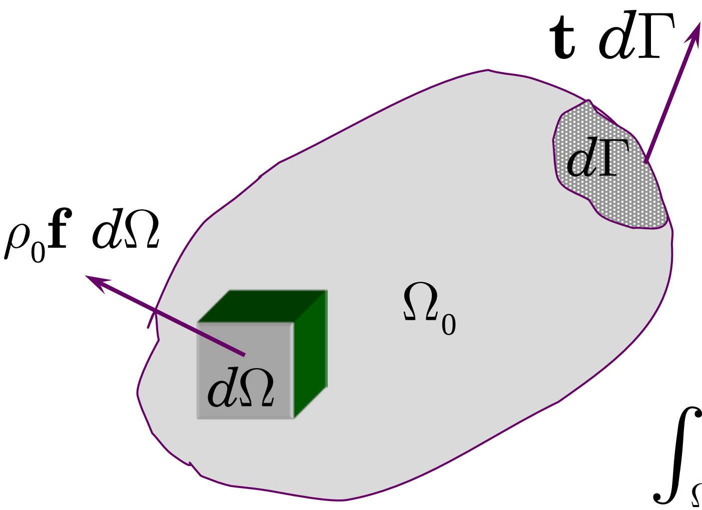
$$\mathbf{t}_{ns} = \mathbf{t}(\hat{\mathbf{n}}) - \mathbf{t}_{nn} = \frac{100}{27}(118\hat{\mathbf{e}}_1 - 16\hat{\mathbf{e}}_2 - 43\hat{\mathbf{e}}_3) \text{ psi}.$$

The magnitudes are

$$|\mathbf{t}_{nn}| = t_{nn} = \frac{2600}{9} = 288.89 \text{ psi}, \quad |\mathbf{t}_{ns}| = t_{ns} = 468.91 \text{ psi}.$$

# Balance of Linear Momentum in the Lagrangian Description

The time rate of change of total linear momentum of a given continuum equals the vector sum of all external forces acting on the continuum. This also known as **Newton's Second Law**.




$$\oint_{\Gamma_0} \mathbf{t}\, d\Gamma + \int_{\Omega_0} \rho_0 \mathbf{f}\, d\Omega = \frac{\partial}{\partial t} \int_{\Omega_0} \rho_0 \mathbf{v}\, d\Omega$$

$$\oint_{\Gamma_0} \hat{\mathbf{n}} \cdot \boldsymbol{\sigma}\, d\Gamma + \int_{\Omega_0} \rho_0 \mathbf{f}\, d\Omega = \int_{\Omega_0} \rho_0 \frac{\partial \mathbf{v}}{\partial t} d\Omega$$

$$\int_{\Omega_0} \left( \nabla \cdot \boldsymbol{\sigma} + \rho_0 \mathbf{f} - \rho_0 \frac{\partial \mathbf{v}}{\partial t} \right) d\Omega = 0$$

**Newton's First Law**. Newton's First Law states that an object will remain at rest or in uniform motion in a straight line unless acted upon by an external force.

# Balance of Linear Momentum
## (continued)

## Vector form of the equation of motion

$$\boxed{\nabla \cdot \boldsymbol{\sigma} + \rho_0 \mathbf{f} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2}}$$

## Cartesian Component Form

$$\left( \hat{\mathbf{e}}_k \frac{\partial}{\partial x_k} \right) \cdot \left( \boldsymbol{\sigma}_{ji} \hat{\mathbf{e}}_j \hat{\mathbf{e}}_i \right) + \rho_0 f_i \hat{\mathbf{e}}_i = \rho_0 \frac{\partial^2 (u_i \hat{\mathbf{e}}_i)}{\partial t^2}$$

$$\frac{\partial \sigma_{ji}}{\partial x_j} \hat{\mathbf{e}}_i + \rho_0 f_i \hat{\mathbf{e}}_i = \rho_0 \frac{\partial v_i}{\partial t} \hat{\mathbf{e}}_i \;\Rightarrow\; \frac{\partial \sigma_{ji}}{\partial x_j} + \rho_0 f_i = \rho \frac{\partial^2 u_i}{\partial t^2}$$

# Balance of Linear Momentum
## (continued)

## Cartesian Component Form (expanded form)

$$\frac{\partial \sigma_{ji}}{\partial x_j} + \rho_0 f_i = \rho_0 \frac{\partial^2 u_i}{\partial t^2}$$

$$\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{21}}{\partial x_2} + \frac{\partial \sigma_{31}}{\partial x_3} + \rho_0 f_1 = \rho_0 \frac{\partial^2 u_1}{\partial t^2}$$

$$\frac{\partial \sigma_{12}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} + \frac{\partial \sigma_{32}}{\partial x_3} + \rho_0 f_2 = \rho_0 \frac{\partial^2 u_2}{\partial t^2}$$

$$\frac{\partial \sigma_{13}}{\partial x_1} + \frac{\partial \sigma_{23}}{\partial x_2} + \frac{\partial \sigma_{33}}{\partial x_3} + \rho_0 f_3 = \rho_0 \frac{\partial^2 u_3}{\partial t^2}$$

# Balance of Angular Momentum

The principle of balance of angular momentum can be stated as: *the time rate of change of the total moment of momentum for a continuum is equal to vector sum of the moments of external forces acting on the continuum*. We assume that there are no body (volume dependent) couples **M**:

$$\lim_{\Delta V \to 0} \Delta \mathbf{M} / \Delta V = \mathbf{0}$$

Then the balance of angular momentum requires

$$\oint_{\Gamma} \mathbf{x} \times \mathbf{t} \, d\Gamma + \int_{\Omega} \mathbf{x} \times \mathbf{f} \, d\Omega = \frac{D}{Dt} \int_{\Omega} \mathbf{x} \times \rho \mathbf{v} \, d\Omega$$

which results in the symmetry of the Cauchy stress tensor:

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}^{\mathrm{T}} \quad \text{or} \quad \sigma_{ij} = \sigma_{ji}$$

# Balance of Energy
## (in spatial description)

The first law of thermodynamics can be stated as: *the time rate of the total energy is equal to the sum of the rate of work done by the external forces and the change of heat content per unit mass*. The second law of thermodynamics provides a restriction on the inter-convertibility of energies (e.g., thermal to mechanical). The first law can be expressed as ($e_c$ is the internal energy density, $g$ is the internal heat generation, and **d** is the symmetric part of the velocity gradient)

$$\frac{d}{dt}\int_\Omega \rho\left(\tfrac{1}{2}\mathbf{v}\cdot\mathbf{v}+e_c\right)d\Omega=\frac{1}{2}\frac{D}{Dt}\int_\Omega \rho\mathbf{v}\cdot\mathbf{v}\,d\Omega+\int_\Omega\left(\boldsymbol{\sigma}:\mathbf{d}-\nabla\cdot\mathbf{q}+g\right)d\Omega$$

which results in $$\boxed{\rho\frac{de_c}{dt}=\boldsymbol{\sigma}:\mathbf{d}-\nabla\cdot\mathbf{q}+g}$$

# SUMMARY

**In these lectures we have covered the following topics with some examples:**

- **Continuum assumption**
- **Kinematics of deformation: Introduced the Green-Lagrange strain tensor**
- **Kinetics: Defined Cauchy stress vector**
- **Cauchy's formula is derived and Cauchy stress tensor is introduced**
- **Balance of linear and angular momenta**
- **Conservation of energy (the first law)**