STUDY AREAS IN COMPOSITES

Characterization of material properties:

Static, dynamic, fatigue, fracture, temperature, moisture, electrical, etc.

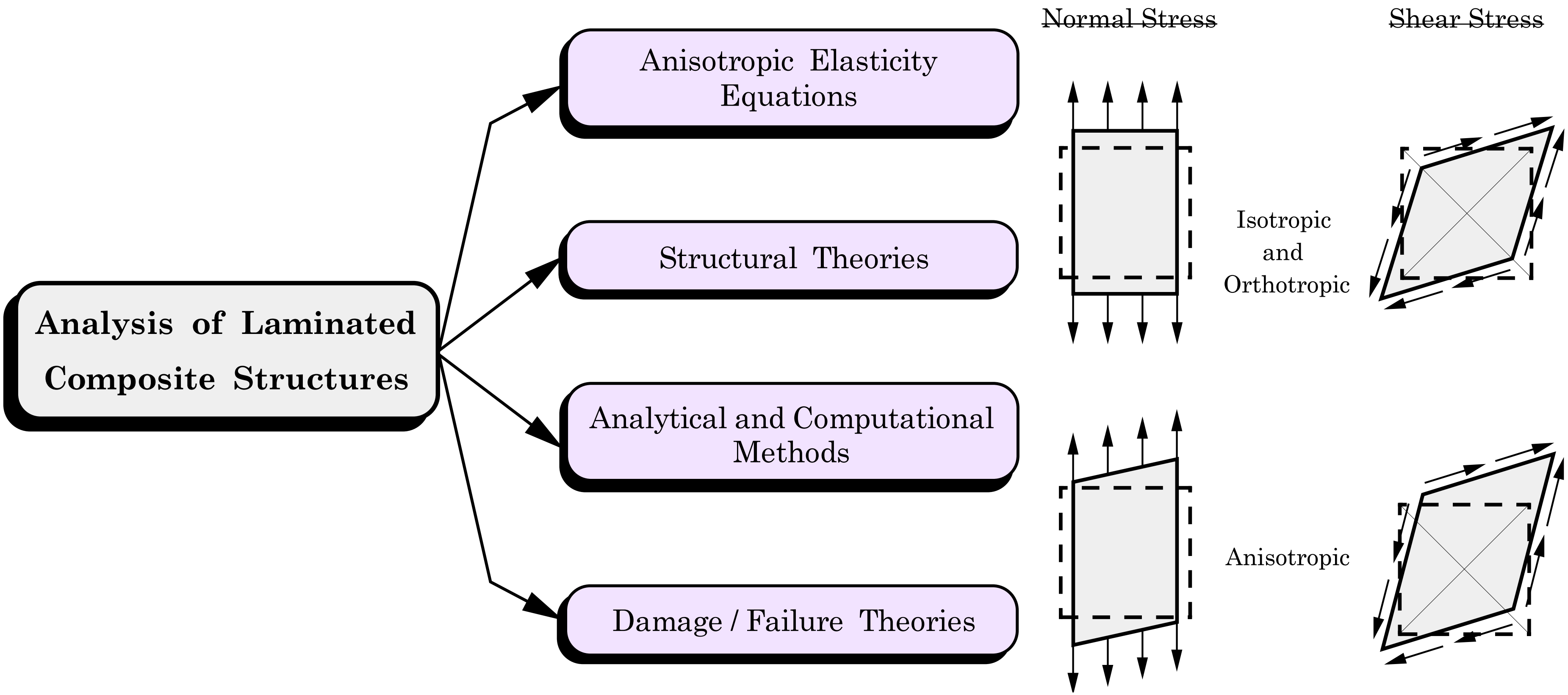
Analyses:

Directional nature of response; different load-response; coupling (shear-extension, bend-twist, bending-extension); many layers versus one.

- Bonded and bolted joints
- Holes in laminates
- Fracture mechanics
- Optimization
- Interlaminar stresses
- Nonlinear material behavior

ANALYSIS OF COMPOSITE STRUCTURES:

The Big Picture



ANALYSIS OF COMPOSITE STRUCTURES:

Sequence of Steps Involved

DEVELOP LAMINIA PROPERTY DATA

↓

LAMINATE ANALYSIS

↓

STRUCTURAL ANALYSIS

Determine failure via

- **Deflections & Stresses**
- **Buckling**
- **Vibrations**

as a linear or nonlinear function of CTE, Moisture, Temperature, Space Environment, Defects, Fracture, Fatigue, Impact, Creep, Strain Rate, Damping, Damage (Wear out)

Joint Analysis

Edge Effect Analysis

Transverse Shear Analysis

Elements of Analysis

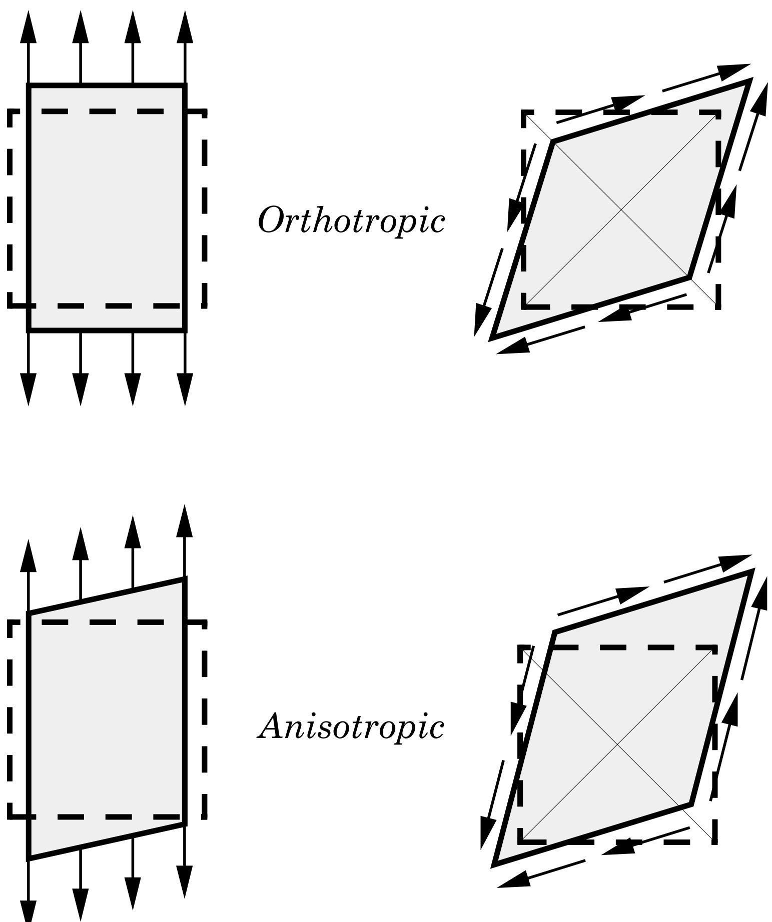
Determine pertinent structural response parameters:

- Deflections
- Stresses
  - Full-field stresses
  - Stresses around cutouts
  - Stresses around defects
- Buckling loads
- Vibration frequencies

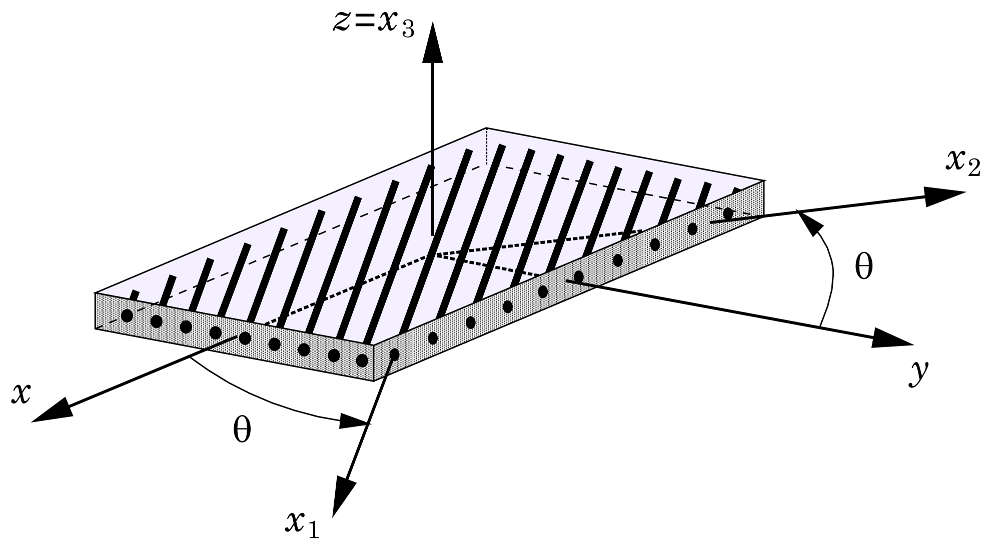
MECHANICAL CHARCATERIZATION

Mechanical Behavior of Isotropic and Orthotropic Materials

~~Normal Stress~~ ~~Shear Stress~~



Transformation of stresses, strains, and material stiffnesses from lamina coordinates (x_1, x_2, x_3) to structural coordinates (x,y,z) is required.



Contracted Notation for Stresses and Strains

Table: Correspondence between the tensor and contracted components of stress and strain.

Stresses		Strains	
Tensor Notation	Contracted Notation	Tensor Notation	Contracted Notation
σ_{11}	σ_1	ε_{11}	ε_1
σ_{22}	σ_2	ε_{22}	ε_2
σ_{33}	σ_3	ε_{33}	ε_3
σ_{23}	σ_4	$2\varepsilon_{23}$	ε_4
σ_{13}	σ_5	$2\varepsilon_{13}$	ε_5
σ_{12}	σ_6	$2\varepsilon_{12}$	ε_6

Strain-Stress Relations:

Generalized Hooke's Law

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix}$$

$$\frac{\nu_{21}}{E_2} = \frac{\nu_{12}}{E_1}; \quad \frac{\nu_{31}}{E_3} = \frac{\nu_{13}}{E_1}; \quad \frac{\nu_{32}}{E_3} = \frac{\nu_{23}}{E_2}$$

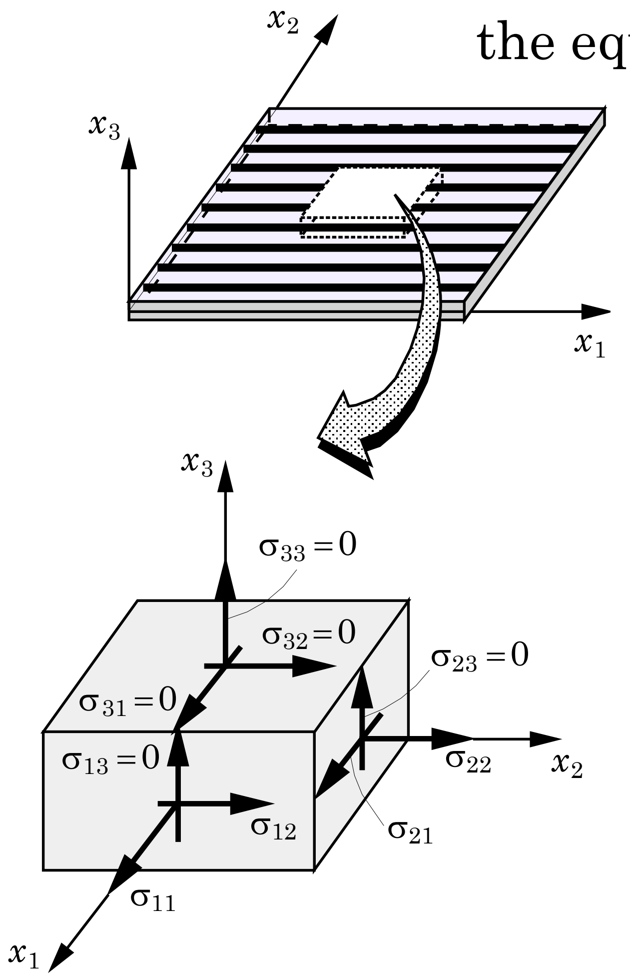
$$\frac{\nu_{ij}}{E_i} = \frac{\nu_{ji}}{E_j} \quad (\text{no sum on } \; i, j)$$

There are nine independent material constants:

$$E_1,\ E_2,\ E_3,\ G_{23},\ G_{13},\ G_{12},\ \nu_{12},\ \nu_{13},\ \nu_{23}$$

Plane-Stress Reduced State

Begin with the **strain-stress** relations and set to the transverse stress components to zero. Then invert the equations to obtain the stress-strain relations.



$$\sigma_3 = 0,\ \sigma_4 = 0,\ \sigma_5 = 0 \quad (\sigma_{33} = 0,\ \sigma_{32} = 0,\ \sigma_{31} = 0)$$

$$\varepsilon_3 = S_{13}\sigma_1 + S_{23}\sigma_2, \quad \varepsilon_4 = 0,\ \varepsilon_5 = 0$$

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix}$$

$$= \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix}$$

Constitutive Equations of the Plane-Stress State

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix}$$

$$Q_{11} = \frac{S_{22}}{S_{11}S_{22} - S_{12}^2} = \frac{E_1}{1 - \nu_{12}\nu_{21}}$$

$$Q_{12} = \frac{S_{12}}{S_{11}S_{22} - S_{12}^2} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}}$$

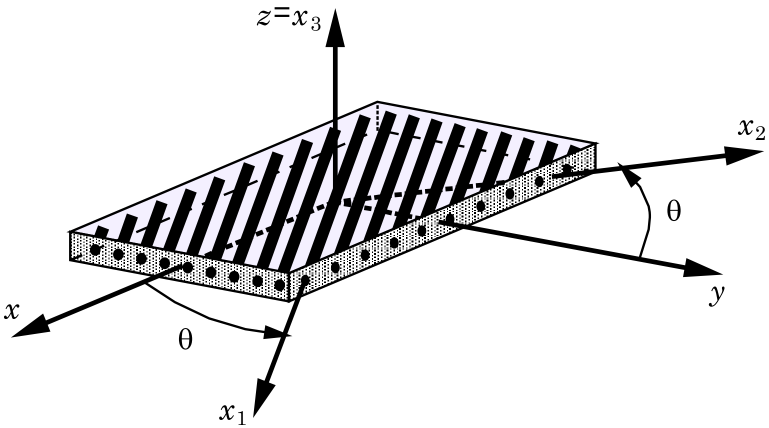
$$Q_{22} = \frac{S_{11}}{S_{11}S_{22} - S_{12}^2} = \frac{E_2}{1 - \nu_{12}\nu_{21}}$$

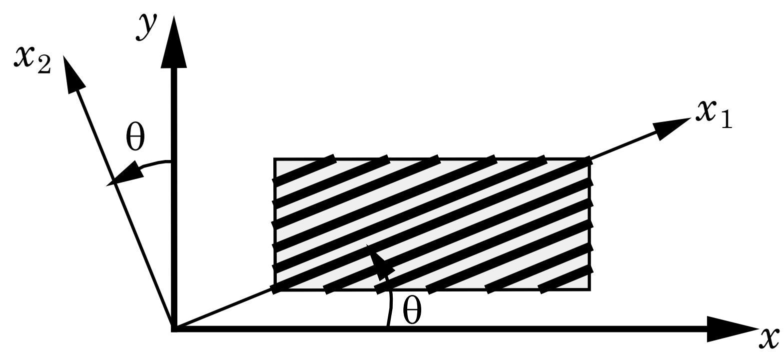
$$Q_{66} = \frac{1}{S_{66}} = G_{12}$$

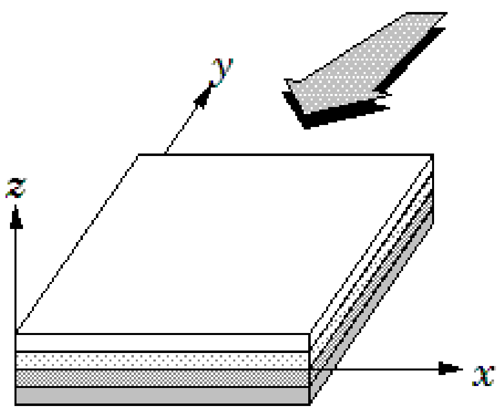
$$E_1, \quad E_2, \quad \nu_{12}, \quad G_{12}$$

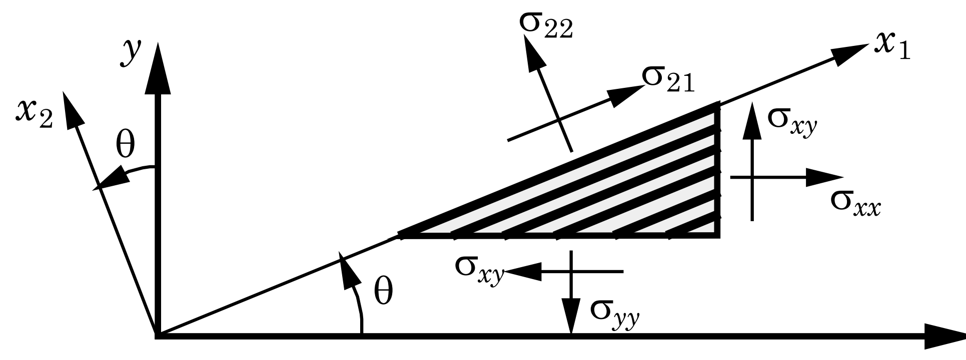
Strain-Stress Relations
in Structural Coordinates

(x_1, x_2, x_3) = Material coordinates **(x, y, z) = Structural coordinates**









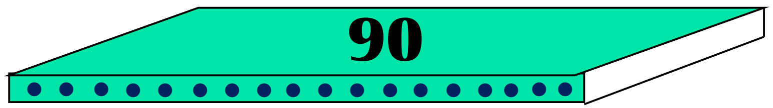
Strain-Stress Relations in Structural Coordinates

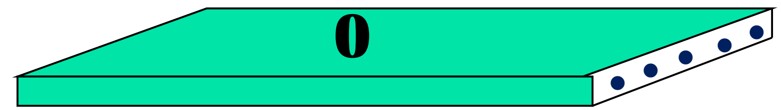
$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{44} & \bar{Q}_{45} \\ \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}$$

$$\begin{aligned}
\bar{Q}_{11} &= Q_{11}\cos^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\ \cos^2\theta + Q_{22}\sin^4\theta \\
\bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})\sin^2\theta\ \cos^2\theta + Q_{12}(\sin^4\theta + \cos^4\theta) \\
\bar{Q}_{22} &= Q_{11}\sin^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\ \cos^2\theta + Q_{22}\cos^4\theta \\
\bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})\sin\theta\ \cos^3\theta + (Q_{12} - Q_{22} + 2Q_{66})\sin^3\theta\ \cos\theta \\
\bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})\sin^3\theta\cos\theta + (Q_{12} - Q_{22} + 2Q_{66})\sin\theta\ \cos^3\theta \\
\bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\sin^2\theta\ \cos^2\theta + Q_{66}(\sin^4\theta + \cos^4\theta) \\
\bar{Q}_{44} &= Q_{44}\cos^2\theta + Q_{55}\sin^2\theta \\
\bar{Q}_{45} &= (Q_{55} - Q_{44})\cos\theta\sin\theta \\
\bar{Q}_{55} &= Q_{55}\cos^2\theta + Q_{44}\sin^2\theta
\end{aligned}$$

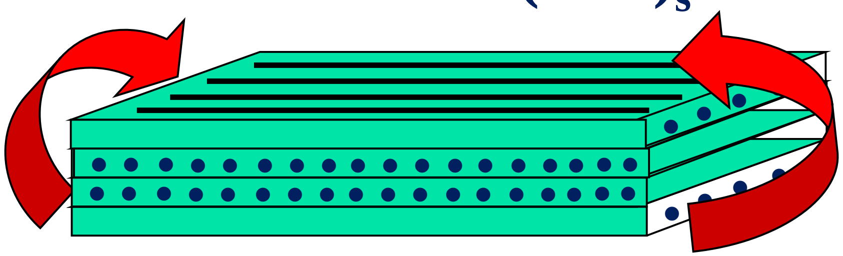
EFFECT OF THE LAMINATION SCHEME on Extensional and Bending Deformation





No change in extensional response

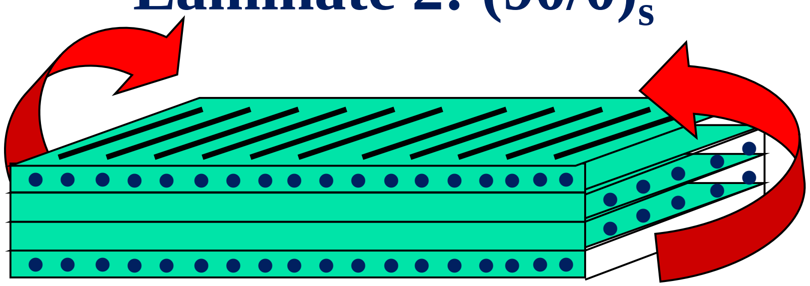
Laminate 1: $(0/90)_s$



$(A_{ij})_1 = (A_{ij})_2$

Stiffer in bending

Laminate 2: $(90/0)_s$



$(D_{ij})_1 > (D_{ij})_2$

Softer in bending