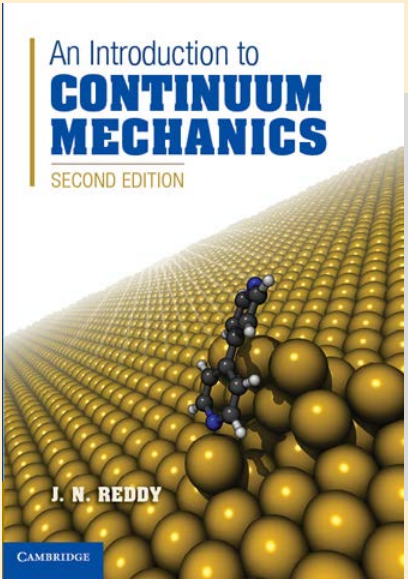
A First Course on
Applied Continuum Mechanics

The contents of the course are continuously updated.

- **J.N. Reddy**, *An Introduction to Continuum Mechanics*, **2nd ed.**, Cambridge University Press, New York, 2013 (*Solution manual is available from the publisher to the course instructors for adopting the book as the primary text book*).
- **J.N. Reddy**, ***Principles of Continuum Mechanics. A Study of Conservation Principles with Applications***, Cambridge University Press, New York, 2010 (**2nd ed**. will appear in 2017). **Translated into French** and published in 2015 by De Boeck Superieur (*Solution manual is available from the publisher to the course instructors for adopting the book as the primary text book*).



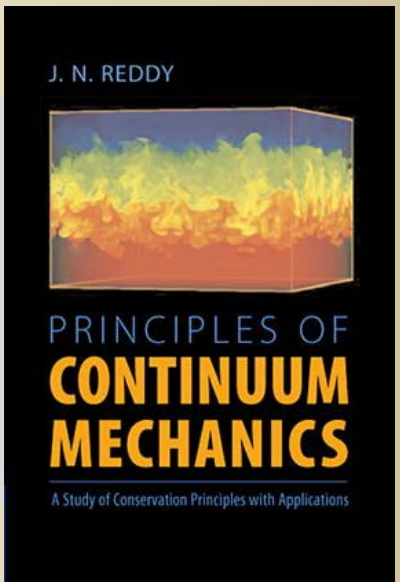
by

J. N. Reddy

e-mail: jnreddy@tamu.edu

Texas A&M University

College Station, TX 77843-3123 USA



CONTENTS OF THE COURSE

GENERAL INTRODUCTION

Scientists and Engineers *formulate*, *analyze*, *test*, *design*, and *manufacture* engineering systems. Formulation involves

- a physical understanding of the system parameters and data,
- knowledge of what needs to be determined through analysis, and
- **developing the pertinent equations governing the system in the context of what needs to be determined.**

The governing equations are developed using certain assumptions concerning the system and appropriate laws of physics: the *conservation of mass*, *balance of momenta*, and *balance of energy* – ***principles of continuum mechanics***

What is Mechanics and a Continuum ?

The subject of **mechanics** deals with the study of deformations and forces in matter, whether it is a solid, liquid, or gas.

In such a study, we make the simplifying assumption, for analytical purposes, that the matter is distributed continuously, without any macroscopic gaps or empty spaces; that is, we disregard the molecular structure of matter.

Such a hypothetical continuous matter is termed a **continuum**. In essence, in a continuum all quantities such as mass density, displacements, velocities, stresses, and so on vary continuously so that their spatial derivatives exist and are continuous.

What is Continuum Mechanics?

For example, mass density ρ (mass per unit volume) of a material at a point is defined as the ratio of the mass Δm of the material to its volume ΔV surrounding the point in the limit that ΔV becomes a value ϵ^3 where ϵ is small compared with the mean distance between molecules:

$$\rho = \lim_{\Delta V \to \epsilon^3} \frac{\Delta m}{\Delta V} \qquad \epsilon \to 0$$

A mathematical study of the mechanics of such an idealized continuum is called

continuum mechanics.

Study of Continuum Mechanics

The subject of **continuum mechanics** deals with the study of deformations and forces in a continuum, whether it is a solid, liquid, or gas.

The **study of continuum mechanics** requires:

- Identify the system or a portion of it that is to be studied.
- Apply a pertinent **laws of physics** to the system to derive the governing equations.
- Analytical formulations require certain mathematical tools, such as **vector and tensor analysis**.

ABOUT THIS COURSE

The primary objectives of this course are:

(1) to study the conservation principles in mechanics of continua and formulate the equations that describe the motion and mechanical behavior of materials, and

(2) to present the applications of these equations to simple problems associated with flows of fluids, conduction of heat, and deformations of solid bodies.

Although the first of these objectives is important, the reason for the formulation of the equations is to gain a quantitative understanding of the behavior of an engineering system.

Mathematics is the language of scientists and engineers

ABOUT THIS COURSE (continued)

The governing equations of a continuous material are nothing but an analytical representation of the global laws of conservation of mass and balance of momenta and energy and the constitutive response of the continuum. Tailoring these equations to particular problems and solving them constitutes the bulk of engineering analysis and design.

The study of motion and deformation of a continuum can be broadly classified into four basic categories:

(1) Kinematics (strain-displacement equations)
(2) Kinetics (balance of linear and angular momentum)
(3) Thermodynamics (first and second laws of thermodynamics)
(4) Constitutive equations (stress--strain relations)

ABOUT THIS COURSE (continued)

Kinematics is the study of geometric changes without consideration of forces causing the deformation.

Kinetics is the study of the equilibrium of forces and moments acting on a continuum, using the principles of balance of linear and angular momentum. This study leads to equations of motion and symmetry of stress tensor in the absence of body couples.

Thermodynamic principles are concerned with the balance of energy and relations among heat, mechanical work, and thermodynamic properties of a continuum.

Constitutive equations describe thermomechanical behavior of the material of the continuum, and they relate the dependent variables introduced in the kinetic description to those introduced in the kinematic and thermodynamic descriptions.