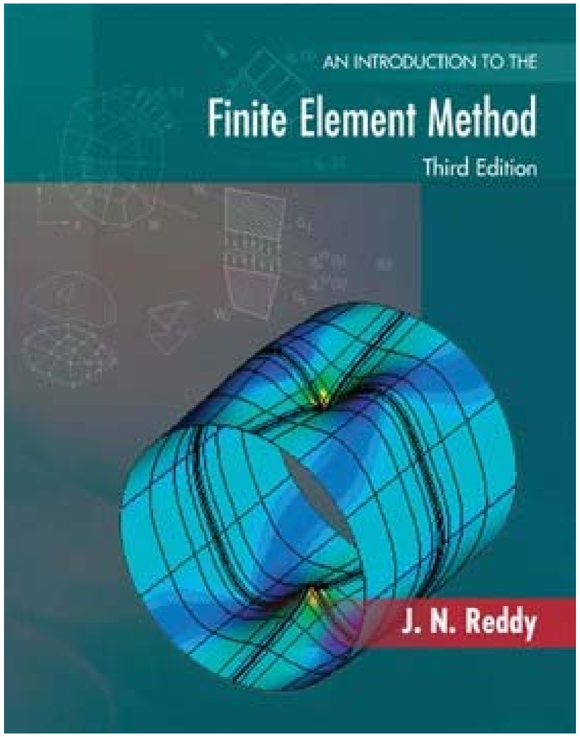
The Finite Element Method

Computer Program FEM1D

Read: **Chapter 7**



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REVIEW OF THE 1-D FE MODELS-1

Governing equation (generalized to include all 1-D problems discussed in Chapters 3-6)

$$c\frac{\partial u}{\partial t}+m\frac{\partial^2 u}{\partial t^2}-\frac{\partial}{\partial x}\left(a\frac{\partial u}{\partial x}\right)+\frac{\partial^2}{\partial x^2}\left(b\frac{\partial^2 u}{\partial x^2}\right)+c_0 u=f(x,t)$$

Weak formulation

$$0=\int_{x_b}^{x_a} v\left[c\frac{\partial u}{\partial t}+m\frac{\partial^2 u}{\partial t^2}-\frac{\partial}{\partial x}\left(a\frac{\partial u}{\partial x}\right)+\frac{\partial^2}{\partial x^2}\left(b\frac{\partial^2 u}{\partial x^2}\right)+c_0 u-f(x,t)\right]dx$$

$$=\int_{x_b}^{x_a}\left[v\left(c\frac{\partial u}{\partial t}+m\frac{\partial^2 u}{\partial t^2}-f\right)+a\frac{\partial v}{\partial x}\frac{\partial u}{\partial x}+b\frac{\partial^2 v}{\partial x^2}\frac{\partial^2 u}{\partial x^2}+c_0 vu\right]dx$$

$$-v(x_a)Q_1-v(x_b)Q_3-\theta(x_a)Q_2-\theta(x_b)Q_4$$

REVIEW OF THE 1-D FE MODELS-2

Semidiscretization

$$\mathbf{C}^e\dot{\mathbf{u}}^e + \mathbf{M}^e\ddot{\mathbf{u}}^e + \mathbf{K}^e\mathbf{u}^e = \mathbf{F}^e$$

$$C_{ij}^e = \int_{x_a}^{x_b} c\,\varphi_i\varphi_j\, dx, \quad M_{ij}^e = \int_{x_a}^{x_b} m\,\varphi_i\varphi_j\, dx$$

$$K_{ij}^e = \int_{\Omega_e}\left(a\frac{d\varphi_i}{dx}\frac{d\varphi_j}{dx} + b\frac{d^2\varphi_i}{dx^2}\frac{d^2\varphi_j}{dx^2} + c_0\varphi_i\varphi_j \right) dx,\ F_i^e = \int_{x_a}^{x_b} f\ \varphi_i\ dx + Q_i$$

Full discretization

$$\hat{\mathbf{K}}^{s+1}\mathbf{u}^{s+1} = \hat{\mathbf{F}}^{s,s+1}$$

$$\hat{\mathbf{K}}^{s+1} = \alpha\Delta t_s\mathbf{K}^{s+1} + \mathbf{C}$$

$$\hat{\mathbf{F}}^{s+1} = \left(\mathbf{C} - (1-\alpha)\Delta t_s\mathbf{K}^{s+1}\right)\mathbf{u}^s + \Delta t_s\left(\alpha\mathbf{F}^{s+1} + (1-\alpha)\mathbf{F}^s\right)$$

$$\hat{\mathbf{K}}^{s+1}\mathbf{u}^{s+1} = \hat{\mathbf{F}}^{s+1}$$

$$\hat{\mathbf{K}}^{s+1} = \mathbf{K}^{s+1} + a_3\mathbf{M}^{s+1} + a_5\mathbf{C}^{s+1}$$

$$\hat{\mathbf{F}}^{s+1} = \mathbf{F}^{s+1} + \mathbf{M}^{s+1}\left(a_3\mathbf{u}^s + a_4\dot{\mathbf{u}}^s + a_5\ddot{\mathbf{u}}^s\right) + \mathbf{C}^{s+1}\left(a_5\mathbf{u}^s + a_6\dot{\mathbf{u}}^s + a_7\ddot{\mathbf{u}}^s\right)$$

NUMERICAL INTEGRATION

$$G_{ij} = \int_{x_a}^{x_b} F_{ij}(x)\,dx \approx \sum_{I=1}^{NPT} F_{ij}(x_I)W_I - \text{General}$$

$$K_{ij} = \int_{-1}^{+1} F_{ij}(\xi)\,d\xi \approx \sum_{I=1}^{NGP} F_{ij}(\xi_I)W_I - \text{Gauss rule}$$

$$NGP = \left[\frac{p+1}{2}\right], \quad \text{Nearest larger integer equal to } (p+1)/2$$

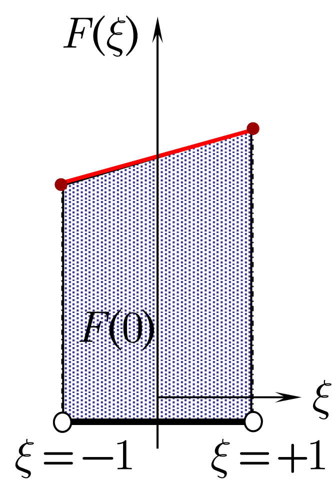
$$p = 1 \quad\Rightarrow \text{NGP} = 1$$

$$p = 2 \text{ or } 3 \Rightarrow \text{NGP} = 2$$

$$p = 4 \text{ or } 5 \Rightarrow \text{NGP} = 3$$

$$I_i = \int_{-1}^{+1} F(\xi)\,d\xi = \frac{h}{2}[F_1 + F_2] = 2\frac{F_1 + F_2}{2}$$

$$= 2 \times F(0) = F(\xi_1)W_1$$



NUMERICAL INTEGRATION-6

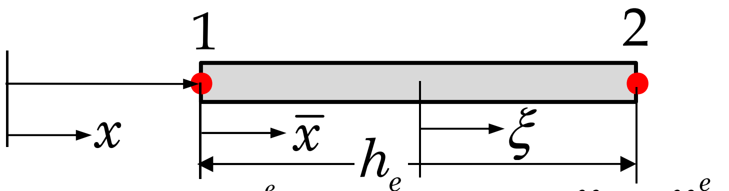
Table 7.1.2: Weights and Gauss points for the Gauss quadrature†.

$$\int_{-1}^{1} F(\xi)d\xi = \sum_{i=1}^{r} F(\xi_i)w_i$$

Points, ξ_i	r	Weights, w_i
0.0000000000	One-point formula	2.0000000000
±0.5773502692	Two-point formula	1.0000000000
0.0000000000	Three-point formula	0.8888888889
±0.7745966692		0.5555555555
±0.3399810435	Four-point formula	0.6521451548
±0.8611363116		0.3478548451
0.0000000000	Five-point formula	0.5688888889
±0.5384693101		0.4786286705
±0.9061798459		0.2369268850
±0.2386191861	Six-point formula	0.4679139346
±0.6612093865		0.3607615730
±0.9324695142		0.1713244924

† Note that $0.57735... = 1/\sqrt{3}$, $0.77459... = \sqrt{3/5}$, $0.888... = 8/9$, $0.555... = 5/9$.

NUMERICAL INTEGRATION-7



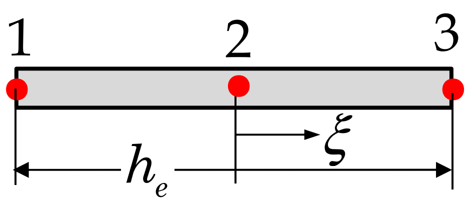
$$x = x_a^e, \quad \bar{x} = 0, \quad \xi = -1$$

$$x = x_b^e, \quad \bar{x} = h_e, \quad \xi = +1$$

$$x = x_a^e + \bar{x}$$
$$\bar{x} = \tfrac{1}{2} h_e (1+\xi)$$
$$x = x_a^e + \tfrac{1}{2} h_e (1+\xi)$$
$$= x_a^e \tfrac{1}{2}(1-\xi) + x_b^e \tfrac{1}{2}(1+\xi)$$

$$\psi_1^e(x) = \frac{x_b^e - x}{h_e}; \quad \psi_1^e(\bar{x}) = 1 - \frac{\bar{x}}{h_e}; \quad \psi_1^e(\xi) = \tfrac{1}{2}(1-\xi)$$

$$\psi_2^e(x) = \frac{x - x_a^e}{h_e}; \quad \psi_2^e(\bar{x}) = \frac{\bar{x}}{h_e}; \quad \psi_2^e(\xi) = \tfrac{1}{2}(1+\xi)$$



$$\psi_1^e(\xi) = -\tfrac{1}{2}\xi(1-\xi), \ \psi_2^e(\xi) = -\tfrac{1}{2}(1+\xi^2), \ \psi_2^e(\xi) = \tfrac{1}{2}\xi(1-\xi)$$

LOGIAL UNITS OF A FEA PROGRAM

PREPROCESSOR

Read

- geometry and material data
- finite element mesh data
- boundary (and initial) conditions

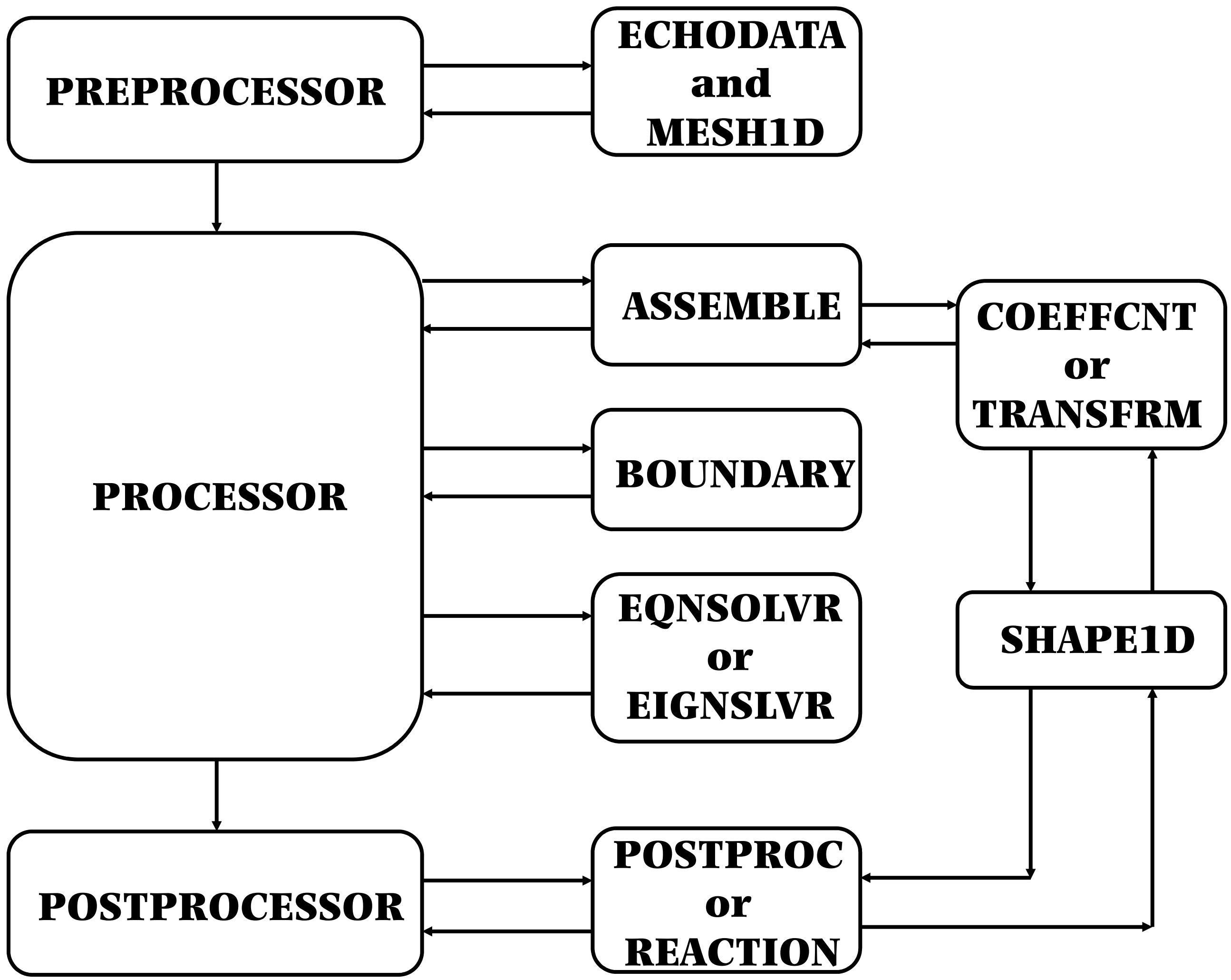
PROCESSOR

- Generate finite element mesh
- Calculate element matrices
- Assemble element equations
- Apply boundary conditions
- Solve the equations
- Check error tolerance and maximum number of allowable iterations

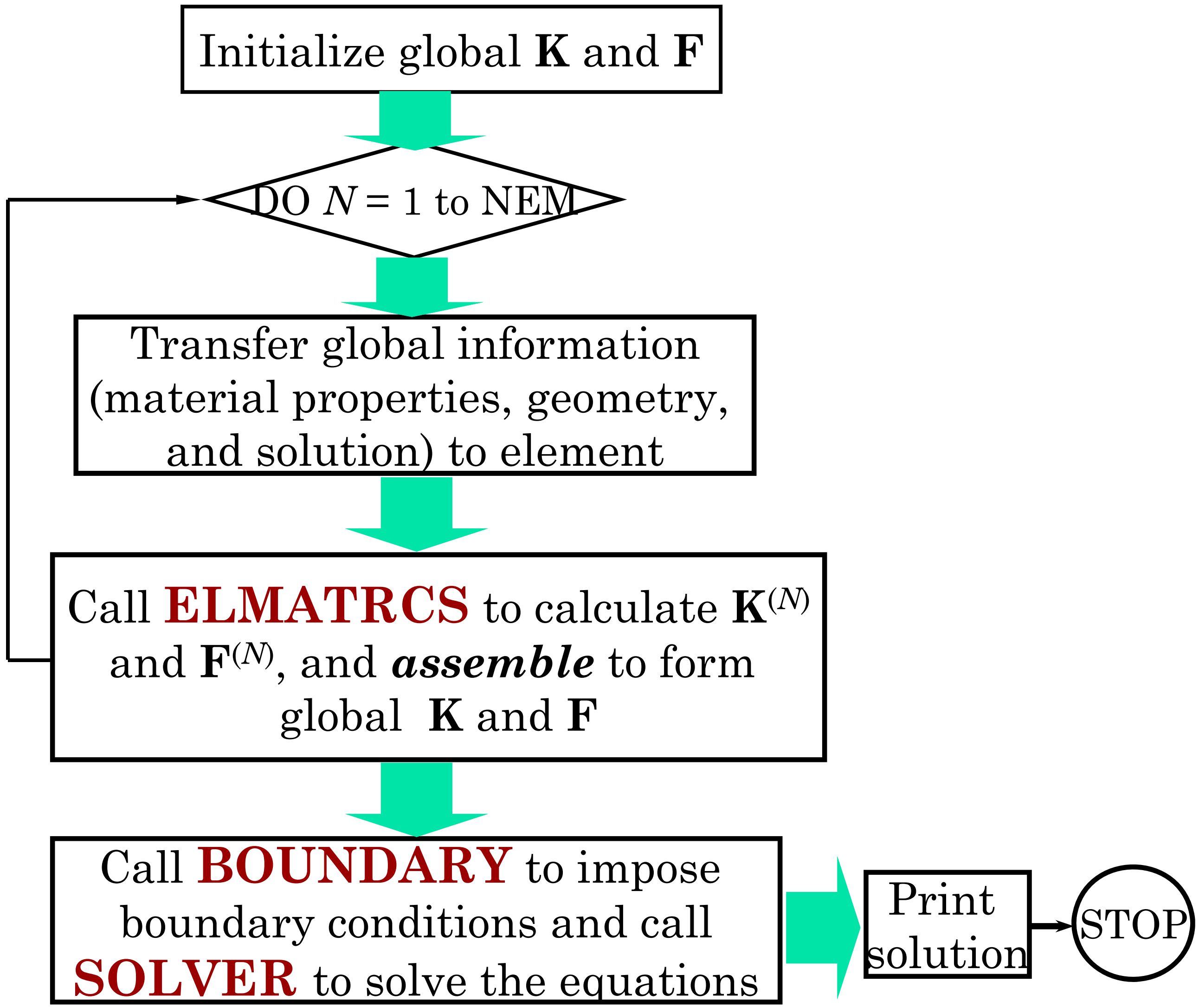
POSTPROCESSOR

- Compute the solution and its derivatives at desired points of the domain
- Print/plot the results

FLOW CHART OF PROGRAM FEM1D



Flow Chart of a Typical PROCESSOR Unit



FINITE ELEMENT PROGRAM FEM1D-1

Variables used in the program

NPE - nodes per element, n

ELX(i) - Global coordinate of the ith node of element e, x_i^e

ELK(i, j) - Element coefficient, K_{ij}^e

ELF(i) - Element coefficient, f_i^e

$AX0, AX1$ – Coefficients in the definition of $a(x)$: $a(x) = AX0 + AX1 * x$

SFL(i) – Element shape (or approximation) funtion, ψ_i^e

DSFL(i) – Derivative of the ith shape function with respect to the local (normalized) coordinate ξ: $\dfrac{d\psi_i}{d\xi}$

GDSFL(i) – Derivative of the ith shape function with respect to the global coordinate x: $\dfrac{d\psi_i}{dx}\left(= \dfrac{d\psi_i}{d\xi}\dfrac{d\xi}{dx} \equiv J^{-1}\dfrac{d\psi_i}{d\xi}\right)$

FINITE ELEMENT PROGRAM FEM1D-2

Numerical Integration

$$x = \sum_{j=1}^{n} x_j^e \psi_j^e(\xi) = \sum_{j=1}^{n} ELX(j) * SFL(j),$$

$$\int_{x_a}^{x_b} \left(\psi_i(x)\psi_j(x) + \frac{d\psi_i}{dx}\frac{d\psi_j}{dx} \right) dx = \int_{-1}^{1} \left(\psi_i(\xi)\psi_j(\xi) + \frac{1}{J_e}\frac{d\psi_i}{d\xi}\frac{1}{J_e}\frac{d\psi_j}{d\xi} \right) J_e\, d\xi$$

$$\approx \sum_{I=1}^{NGP} \hat{F}_{ij}(\xi_I) w_I = \sum_{I=1}^{NGP} \hat{F}_{ij}(\text{GAUSPT}(I,NGP)) * GAUSWT(J,NGP)$$

$$\hat{F}_{ij}(\xi) = \left(\psi_i(\xi)\psi_j(\xi) + \frac{1}{J_e}\frac{d\psi_i}{d\xi}\frac{1}{J_e}\frac{d\psi_j}{d\xi} \right) J_e,$$

$$\frac{d\psi_i}{dx} = \frac{d\xi}{dx}\frac{d\psi_i}{d\xi} = \frac{1}{J_e}\frac{d\psi_i}{d\xi},\quad dx = J_e d\xi \quad \text{or} \quad J_e = \frac{dx}{d\xi} = 0.5\,h_e$$

$GAUSPT(I,j) - I$thGauss point, ξ_I, for the j-point Gauss rule

$GAUSWT(I,j) - I$thGauss weight, w_I, for the j-point Gauss rule

IMPOSOITION OF BOUNDARY CONDITIONS

NSPV – Number of specified primary variables of the problem.

ISPV(I,J) – Array containing the information about the global node number and the local degree of freedom that is specified.

ISPV(I,1) – For the Ith boundary condition, the global node number at which the BC is specified.

ISPV(I,2) – For the Ith boundary condition, the local degree of freedom that is specified.

VSPV(I) – The specified value of the deg. of freedom.

Similar meaning for NSSV, ISSV, and VSSV for specified secondary variables

IMPOSOITION OF BOUNDARY CONDITIONS

NDF= number of primary degrees of freedom at a node

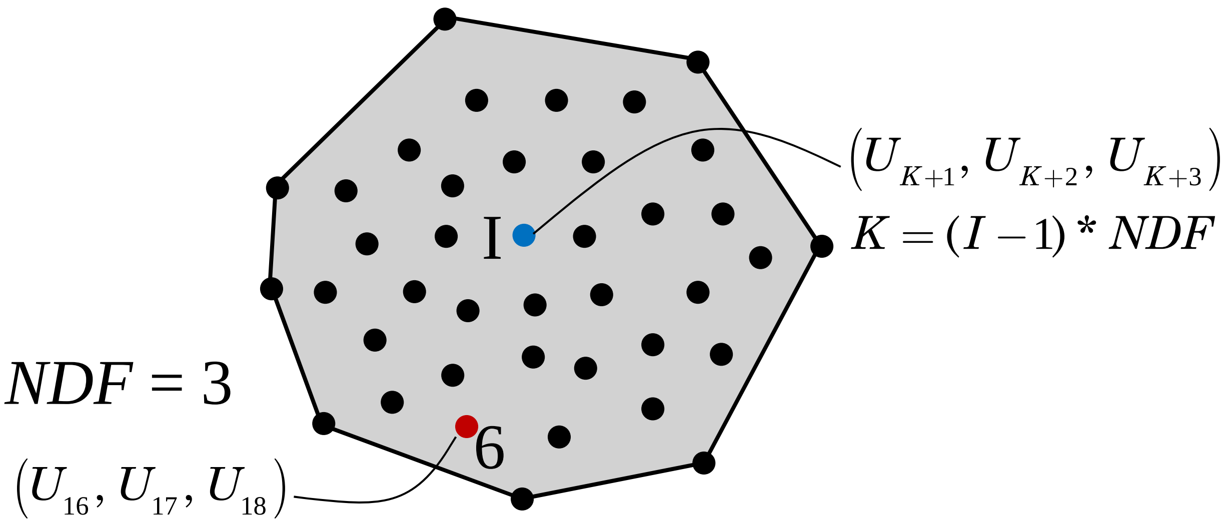


Table 7.3.2: Description of the input variables to FEM1D (revised from the book)-1

*** Data Card 1: TITLE**

*** Data Card 2: MODEL, NTYPE, ITEM**

MODEL=1, NTYPE=0: A problem of MODEL EQUATION 1
MODEL=1, NTYPE=1: A circular DISK (PLANE STRESS)
MODEL=1, NTYPE>1: A circular DISK (PLANE STRAIN)
MODEL=2, NTYPE=0: A Timoshenko BEAM (RIE#) problem
MODEL=2, NTYPE=1: A Timoshenko PLATE (RIE) problem
MODEL=2, NTYPE=2: A Timoshenko BEAM (CIE##) problem
MODEL=2, NTYPE>2: A Timoshenko PLATE (CIE) problem
MODEL=3, NTYPE=0: A Euler-Bernoulli BEAM problem
MODEL=3, NTYPE>0: A Euler-Bernoulli Circular plate
MODEL=4, NTYPE=0: A plane TRUSS problem
MODEL=4, NTYPE=1: A Euler-Bernoulli FRAME problem
MODEL=4, NTYPE=2: A Timoshenko (CIE) FRAME problem
ITEM=0, Steady-state solution
ITEM=1, Transient analysis of PARABOLIC equations
ITEM=2, Transient analysis of HYPERBOLIC equations
ITEM=3, Eigenvalue analysis

*** Data Card 3: IELEM, NEM**

IELEM=0, Hermite cubic finite element
IELEM=1, Linear Lagrange finite element
IELEM=2, Quadratic Lagrange finite element

Table 7.3.2: Description of the input variables to the program FEM1D -2

*** Data Card 4: ICONT, NPRNT**

ICONT=1, Data (AX,BX,CX,FX and mesh) is continuous

ICONT=0, Data is element dependent

NPRNT=0, Not print element or global matrices
but postprocess the solution and print

NPRNT=1, Print Element 1 coefficient matrices only
but postprocess the solution and print

NPRNT=2, Print Element 1 and global matrices but
NOT postprocess the solution

NPRNT>2, Not print element or global matrices and
NOT postprocess the solution

SKIP Cards 5 through 15 for TRUSS/FRAME problems (MODEL = 4), and read Cards 5 through 15 only if MODEL.NE.4.

*** Data Card 5: DX(I)**

Array of element lengths. DX(1) denotes the global coordinate of Node 1 of the mesh; DX(I) (I=2,NEM+1) denotes the length of the (I-1)st element. Here NEM denotes the number of elements in the mesh.

Table 7.3.2: Description of the input variables to the program FEM1D -3

Cards 6 through 9 define the coefficients in the model equations. All coefficients are expressed in terms of GLOBAL coordinate x. See Table 7.3 for the meaning of the coefficients, especially for deformation of circular plates and Timoshenko elements.

*** Data Card 6: AX0, AX1, AX2**

*** Data Card 7: BX0, BX1**

*** Data Card 8: CX0, CX1**

SKIP Card 9 for eigenvalue problems (i.e. ITEM=3)

*** Data Card 9: FX0, FX1, FX2**

*** Data Card 10: NNM**

Table 7.3.2: Description of the input variables to the program FEM1D -4

SKIP Cards 11 through 16 if data is continuous (ICONT.NE.0). Cards 11-14 are read for each element (i.e., NEM times). All coefficients are expressed in terms of the GLOBAL coordinates.

*** Data Card 11: DCAX(n,i), i=1,2,3**

*** Data Card 12: DCBX(n,i), i=1,2**

*** Data Card 13: DCCX(n,i), i=1,2**

*** Data Card 14: DCFX(n,i), i=1,2,3**

READ Cards 15 through 20 for TRUSS/FRAME problems (MODEL = 4).
SKIP Cards 15 through 20 if MODEL.NE.4 ___________________

*** Data Card 15: NNM**

SKIP Cards 16 through 18 for TRUSS problems (NTYPE = 0). Cards 16 through 18 are read for each element, i.e., NEM times _________

Table 7.3.2: Description of the input variables to the program FEM1D -5

*** Data Card 16: PR, SE, SL, SA, SI, CS, SN**

PR - Poisson's ratio of the material#
SE - Young's modulus of the material
SL - Length of the element
SA - Cross-sectional area of the element
SI - Moment of inertia of the element
CS - Cosine of the angle of orientation of the element
SN - Sine of the angle of orientation of the element; the angle is measured counter-clock-wise from x axis

*** Data Card 17: HF, VF, PF, XB, CNT, SNT**

HF - Intensity of the horizontal distributed force
VF - Intensity of the transversely distributed force
PF - Point load on the element
XB - Distance from node 1, along the length of the element, to the point of load application, PF
CNT - Cosine of the angle of orientation of the load PF
SNT - Sine of the angle of orientation of the load PF; the angle is measured counter-clock-wise from x axis

Table 7.3.2: Description of the input variables to the program FEM1D -6

*** Data Card 18: NOD**

READ Cards 19 and 20 only for TRUSS problems (NTYPE = 0). Cards 19 and 20 are read for each element; i.e. NEM times _______________

*** Data Card 19: SE, SL, SA, CS, SN**

SE - Young's modulus of the material
SL - Length of the element
SA - Cross-ectional area of the element
CS - Cosine of the angle of orientation of the element
SN - Sine of the angle of orientation of the element
Angle is measured counter-clock-wise from x axis
HF - Intensity of the horizontal distributed force

*** Data Card 20: NOD**

*** Data Card 21: NCON**

Table 7.3.2: Description of the input variables to the program FEM1D -7

SKIP Card 22 if <u>no</u> constraint conditions are specified (NCON = 0). Repeat Card 22 NCON times:____

*** Data Card 22: ICON (i), VCON(i)**

*** Data Card 23: NSPV**

SKIP Card 24 if no primary variables is specified (NSPV=0). Repeat Card 24 NSPV times ____________________________________

*** Data Card 24: ISPV(I,1), ISPV(I,2), VSPV(I) (I = 1 to NSPV)**

ISPV(I,1) - Node number at which the PV is specified
ISPV(I,2) - Specified local primary DOF at the node
VSPV(I) - Specified value of the primary variable (PV)
(will not read for eigenvalue problems)

SKIP Card 25 for eigenvalue problems (i.e. when ITEM=3) __________

*** Data Card 25: NSSV**

Table 7.3.2: Description of the input variables to the program FEM1D -8

SKIP Card 26 if no secondary variable is specified (NSSV=0). Repeat Card 26 NSSV times ____

*** Data Card 26: ISSV(I,1), ISSV(I,2), VSSV(I) (I = 1 to NSSV)**

ISSV(I,1) - Node number at which the SV is specified
ISSV(I,2) - Specified local secondary DOF at the node
VSSV(I) - Specified value of the secondary variable (PV)

*** Data Card 27: NNBC**

SKIP Card 28 if no mixed boundary condition is specified (NNBC=0). The mixed boundary condition is assumed to be of the form: SV+VNBC *(PV-UREF) = 0. Repeat Card 28 NNBC times ________________________

*** Data Card 28: INBC(I,1), INBC(I,2), VNBC(I), UREF(I) (I = 1 to NNBC)**

INBC(I,1) - Node number at which the mixed B.C. is specified
INBC(I,2) - Local DOF of the PV and SV at the node
VNBC(I) - Value of the coefficient of the PV in the B.C.
UREF(I) - Reference value of the PV

Table 7.3.2: Description of the input variables to the program FEM1D -9

*** Data Card 29: NMPC**

SKIP Card 30 if no multipoint constraints are specified (NMPC=0).
Repeat Card 30 NMPC times ___________________________

*** Data Card 30: I=1, NMPC**
(IMC1(I,J),J=1,2),(IMC2(I,J),J=1,2),(VMPC(I,J),J=1,4)

SKIP Card 31 if ITEM=0 (read for time-dependent and eigenvalue problems)

*** Data Card 31: CT0, CT1**

CT0 - Constant part of CT = CT0 + CT1*X
CT1 - Linear part of CT = CT0 + CT1*X

SKIP remaining cards if steady-state or eigenvalue analysis is to be performed (ITEM=0 or ITEM=3) ____________________

*** Data Card 32: DT, ALFA, GAMA**

DT - Time increment (uniform)
ALFA - Parameter in the time approximation scheme
BETA - Parameter in the time approximation scheme

Table 7.3.2: Description of the input variables to the program FEM1D -10

*** Data Card 33: INCOND, NTIME, INTVL**

INCOND- Indicator for initial conditions:
INCOND=0, Homogeneous (zero) initial conditions
INCOND>0, Nonhomogeneous initial conditions
NTIME - Number of time steps for which solution is sought
INTVL - Time step intervals at which solution is to printed

SKIP Cards 34 and 35 if initial conditions are zero (INCOND=0) ___

*** Data Card 34: GU0(I)**

{GU0} - Array of initial values of the primary variables

SKIP Card 34 for parabolic equations (ITEM=1) ________________

*** Data Card 35: GU1(I)**

{GU1} - Array of initial values of the first time-derivatives of the primary variables

TO RUN THE EXECUTABLE PROGRAM FEM1DF15.EXE

Executable Computer Programs from the book, *An Introduction to the Finite Element Method* by J. N. Reddy, 3rd ed., McGraw--Hill, 2006.

Notes:

Programs FEM1D and FEM2D are a revised versions of the programs from the second edition of the book. The revisions are minor. Both FEM1D and FEM2D were compiled using the Microsoft Fortran compiler, with a fixed array dimensions. Hence, only a limited size problem can be analyzed using the compiled versions of the programs. The programs were compiled with a maximum number of degrees of freedom of 2,500. If your computer has the storage and you have the source programs, you may recompile the program after changing the DIMENSION statements in the programs.

TO RUN THE EXECUTABLE PROGRAM
FEM1D.EXE

To run a program on a PC, the files should be downloaded to your PC into a folder (say, *FEM_Reddy*). ***The user is required to prepare a data file for each problem*** he or she wants to solve, using the instructions in the book (use Table 7.3.2 for FEM1D and Table 13.4.1 for FEM2D). Most errors are mistakes made in the preparation of the data files. Therefore, you must check the data files when you see 'run-time error' message or the program is not executed (by returning just the echo of the input data file). All files should be in the same folder where FEM1D.EXE and FEM2D.EXE are placed.

To run the program:

- Double click on the executable file (FEM1D.EXE or FEM2D.EXE).
  A window (called *Command Prompt* window) will pop open (with white letter and black background). It will read

  File name missing or blank – please enter file name
  UNIT 5?

TO RUN THE EXECUTABLE PROGRAM FEM1D.EXE

- Type the input file name (with its extension) and press *Enter*. For example, if the file name you prepared is labeled as **Prob1.inp**, you may type it as **prob1.inp** (not case sensitive). The Command Prompt window will now display

  File name missing or blank – please enter file name
  UNIT 6?

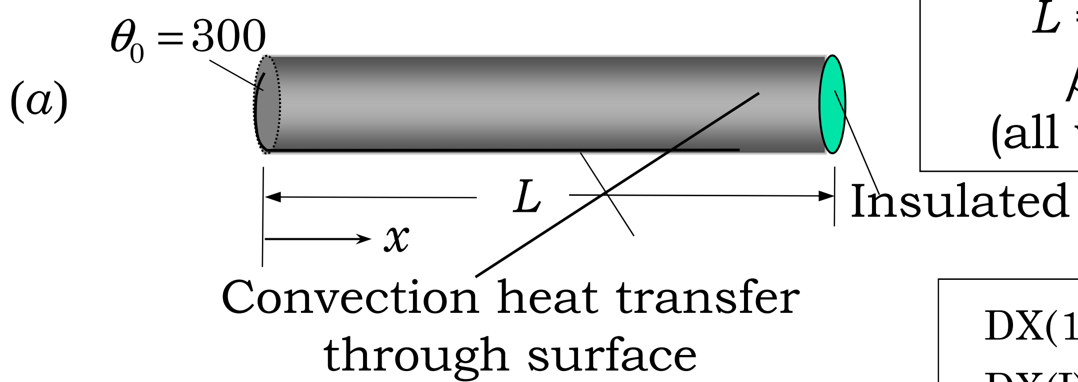
- Type the output file name (with its extension) and press *Enter*. For example, if you want the computer to return the output to file **Prob1.out**, you may type it as **prob1.out**. Note that you do not prepare this file; it will be created by the computer. The Command Prompt window will disappear, indicating that it has taken the data file and executed the program. You will find the file **prob1.out** in the same folder where you are running the program. Depending on the results you see, you may have to correct the data file.

EXAMPLE PROBLEMS FOR PROGRAM FEM1D

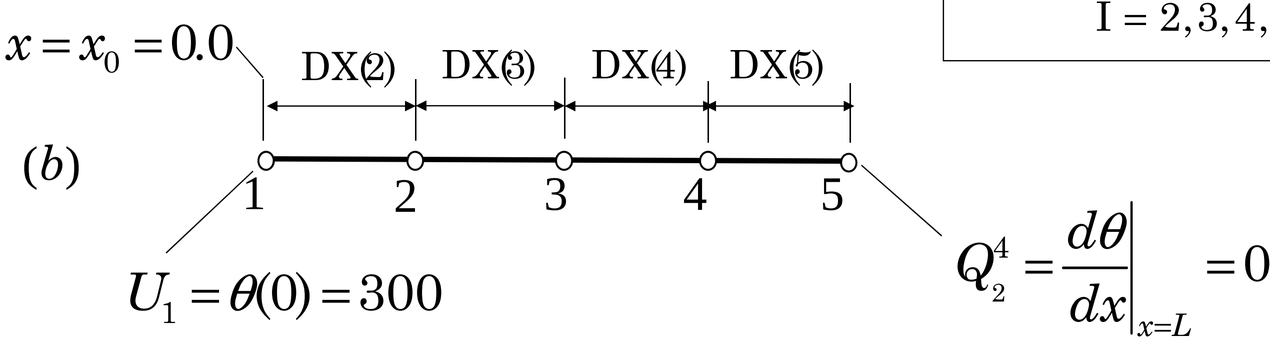
Example 1: One-Dimensional Heat transfer in a Rod

$$-\frac{d^2\theta}{dx^2} + c\,\theta = 0, \quad 0 < x < L \qquad c = m^2 = \frac{\beta P}{Ak} = 400$$

$L = 0.05,\ c = 400,$
$\beta = 100,\ k = 50$
(all with proper units)



$\mathrm{DX}(1) = x_0 = 0.0$
$\mathrm{DX(I)} = h = 0.0125$
$\mathrm{I} = 2,3,4,5$

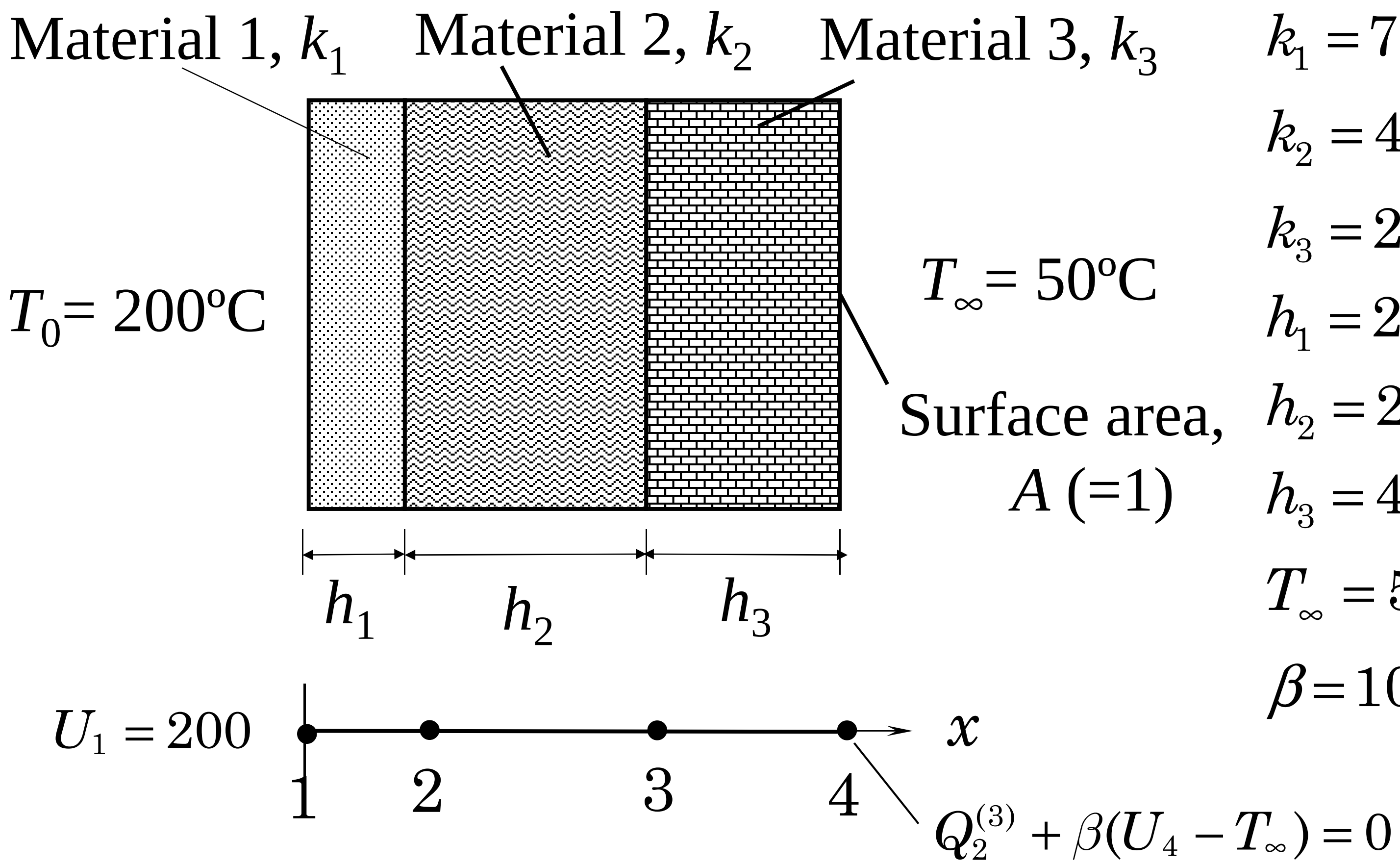


One-Dimensional Heat transfer in a Rod

Example 1: Heat transfer in a rod (4 linear elements)

1 0 0	MODEL, NTYPE, ITEM
1 4	IELEM, NEM
1 1	ICONT, NPRNT
0.0 0.0125 0.0125 0.0125 0.0125	DX(1)=X0; DX(2), etc.
1.0 0.0 0.0	AX0, AX1, AX2
0.0 0.0	BX0, BX1
400.0 0.0	CX0, CX1
0.0 0.0 0.0	FX0, FX1, FX2
1	NSPV
1 1 300.0	ISPV(1,1), ISPV(1,2), VSPV(1)
0	NSSV
0	NNBC
0	NMPC

Example 2: 1-D Heat transfer in a composite wall



$k_1 = 70 \quad W/(m \cdot {}^\circ C)$

$k_2 = 40 \, W/(m \cdot {}^\circ C),$

$k_3 = 20 \quad W/(m \cdot {}^\circ C)$

$h_1 = 2 \, cm$

$h_2 = 2.5 \, cm$

$h_3 = 4 \, cm$

$T_\infty = 50^\circ C$

$\beta = 10 \, W/(m^2 \cdot {}^\circ C)$

1-D Heat transfer in a composite wall

Example 2: Heat transfer in a composite wall

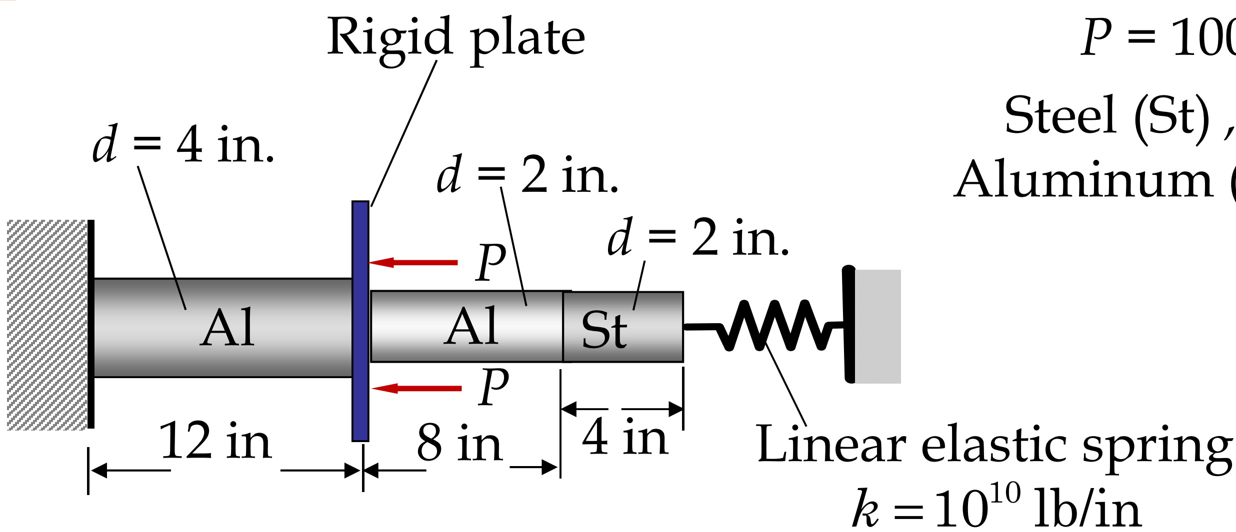
1 0 0	MODEL, NTYPE, ITEM	
1 3	IELEM, NEM	
0 1	ICONT, NPRNT	
0.0 0.02 0.025 0.04	DX(I)	
70.0 0.0 0.0	AX0, AX1, AX2	Data for Element 1
0.0 0.0	BX0, BX1	
0.0 0.0	CX0, CX1	
0.0 0.0 0.0	FX0,FX1,FX2	
40.0 0.0 0.0	AX0, AX1, AX2	Data for Element 2
0.0 0.0	BX0, BX1	
0.0 0.0	CX0, CX1	
0.0 0.0 0.0	FX0,FX1,FX2	

1-D Heat transfer in a composite wall

20.0	0.0	0.0		AX0, AX1, AX2	Data for Element 3
0.0	0.0			BX0, BX1	
0.0	0.0			CX0, CX1	
0.0	0.0	0.0		FX0,FX1,FX2	
1				NSPV	
1	1	200.0		ISPV(1,1), ISPV(1,2), VSPV(1)	
0				NSSV	
1				NNBC	
4	1	10.0	50.0	INBC(1,1),INBC(1,2),VNBC(1),UREF(1)	
0				NMPC	

VNBC(1) = β , UREF(1) = T_∞

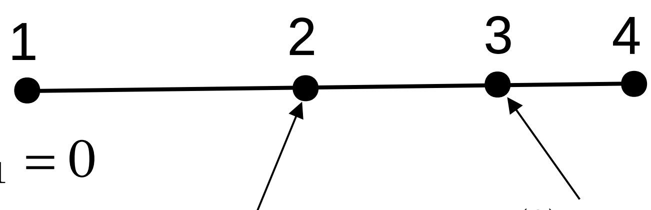
Example 3: Axial deformation of a composite bar



P = 100 kips = 10^5 lb

Steel (St) , $E_s = 30 \times 10^6$ psi

Aluminum (Al), $E_a = 10 \times 10^6$ psi



$$Q_2^{(3)} + kU_4 = 0$$

$$U_1 = 0$$

$$Q_2^{(1)} + Q_1^{(2)} = -2P \qquad Q_2^{(2)} + Q_1^{(3)} = 0$$

Example 3: Axial deformation of a composite bar

Example 3: Axial deformation of a composite bar

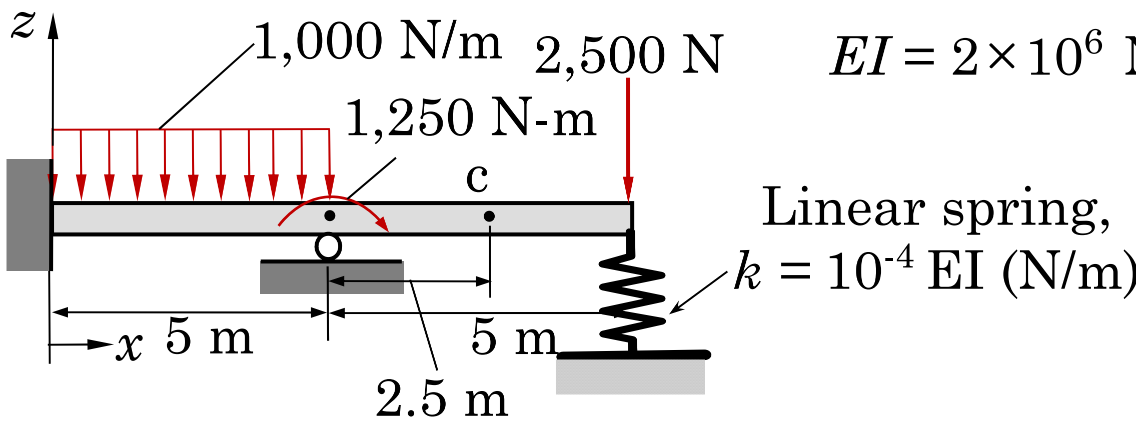
1 0 0	MODEL, NTYPE, ITEM	
1 3	IELEM, NEM	
0 2	ICONT, NPRNT	
0.0 12.0 8.0 4.0	DX(I)	
12.56637E07 0.0 0.0	AX0, AX1, AX2	Element 1
0.0 0.0	BX0, BX1	Element 1
0.0 0.0	CX0, CX1	Element 1
0.0 0.0 0.0	FX0, FX1, FX2	Element 1
3.141593E07 0.0 0.0	AX0, AX1, AX2	Element 2
0.0 0.0	BX0, BX1	Element 2
0.0 0.0	CX0, CX1	Element 2
0.0 0.0 0.0	FX0, FX1, FX2	Element 2

Example 3: Axial deformation of a composite bar

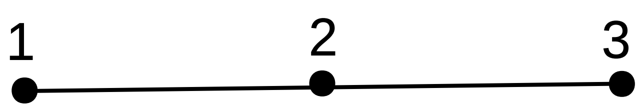
9.424778E07 0.0 0.0	AX0, AX1, AX2	Element 3
0.0 0.0	BX0, BX1	
0.0 0.0	CX0, CX1	
0.0 0.0 0.0	FX0, FX1, FX2	

1	NSPV
1 1 0.0	ISPV(1,1),ISPV(1,2),VSPV(1)
1	NSSV
2 1 –2.0E05	ISSV(1,1),ISSV(1,2),VSSV(1)
1	NNBC
4 1 1.0E10 0.0	INBC(1,1),INBC(1,2),VNBC(1),UREF(1)
0	NMPC

Example 4: Bending of a Beam



$EI = 2\times10^6$ N-m^2



$U_1 = 0$

$U_2 = 0$

$U_3 = 0$

$Q_4^{(1)} + Q_2^{(2)} = 1{,}250$

$Q_3^{(2)} + kU_5 = -2{,}500$

$Q_4^{(2)} = 0$

Example 4: Bending of a beam

Example 4: Clamped and Spring-supported Beam (EBT)

3 0 0	MODEL, NTYPE, ITEM	
0 2	IELEM, NEM	
0 1	ICONT, NPRNT	
0.0 5.0 5.0	DX(I)	
0.0 0.0 0.0	AX0, AX1, AX2	Data for Element 1
2.0E6 0.0	BX0, BX1	
0.0 0.0	CX0, CX1	
–1.0E3 0.0 0.0	FX0, FX1, FX2	
0.0 0.0 0.0	AX0, AX1, AX2	Data for Element 2
2.0E6 0.0	BX0, BX1	
0.0 0.0	CX0, CX1	
0.0 0.0 0.0	FX0, FX1, FX2	

Example 4: Bending of a beam

3	NSPV
1 1 0.0	ISPV(1,1), ISPV(1,2), VSPV(1)
1 2 0.0	ISPV(2,1), ISPV(2,2), VSPV(2)
2 1 0.0	ISPV(3,1), ISPV(3,2), VSPV(3)
2	NSSV
2 2 1250.0	ISSV(1,1), ISSV(1,2), VSSV(1)
3 1 -2500.0	ISSV(2,1), ISSV(2,2), VSSV(2)
1	NNBC (with transverse spring)
3 1 2.0E02 0.0	INBC(1,1),INBC(1,2),VNBC(1),UREF(1)
0	NMPC

SUMMARY

We have discussed the following topics in this presentation:

- **Review of FE Models of Chapters 3 – 6**
- **Numerical integration in 1-D**
- **Logical units of a FEA program**
- **Flow chart of a typical processor unit**
- **Element calculations**
- **Computer program FEM1D**
- **Input data to FEM1D**
- **Example problems for FEM1D**
- **Summary**