Computational Modeling of Polymer Composites

A Study of Creep and Environmental Effects

Dedication

To Our Mothers

Computational Modeling of Polymer Composites

A Study of Creep and Environmental Effects

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Preface

Most significant developments in engineering in the last three decades have been in the area of materials modeling. Foremost among these developments have been the development of polymeric materials and their use in smart materials and structures, functionally graded materials (FGMs), composites, and nanoscience and technology – each topic deserves to be treated in a monograph by itself.

The focus for the present book is the education of the individual who is interested in gaining a good understanding of the theories and associated finite element models of elastic and viscoelastic response of polymers and polymer composites. The subject is truly an interdisciplinary one, where chemists, material scientists, chemical engineers, mechanical engineers, and structural engineers contribute to the overall product. The courses offered at universities and the books published on polymer composites are of three types: material science, mechanics, and numerical simulations. The present book belongs to the *computational mechanics* category in that it covers both mechanics aspects as well as computational aspects.

The motivation for the present book has come from many years of the authors' research and teaching in composite materials and from the fact there does not exist a book that covers computational modeling of polymers and polymeric composites. The book is largely based on the authors' original works in the subject area over the last three decades.

Some mathematical preliminaries, equations of anisotropic elasticity, and virtual work principles and variational methods are reviewed in Chapter 1. A reader who has a knowledge of these topics may skip this chapter and go directly to Chapter 2, where an introduction to the finite element method is presented.

The primary journey of the book begins with Chapter 3, where finite element analysis of viscoelastic materials is presented. Both linear and nonlinear viscoelastic models are considered. Chapter 4 is dedicated to diffusion process in polymers and polymer matrix composites. Chapter 5 covers finite element models of viscoelastic materials. Both uniaxial and multiaxial cases are considered, and delayed failure is discussed. In Chapter 6, finite element analysis of nonlinear Fickean diffusion process in polymers is studied, while non-Fickean diffusion of polymers is discussed in Chapter 7.

Chapter 8 is devoted to a study of coupled hygrothermal cohesive layer model for simulating debond growth in bimaterial interfaces. A viscoelastic cohesive layer model for prediction of interlaminar shear strength of carbon/epoxy composites is presented in Chapter 9. Finally, Chapter 10 deals with a multi-scale viscoelastic cohesive layer model for predicting delamination in high temperature polymer composites.

The book is suitable as a reference for engineers and scientists working in industry and academia, and it can be used as a textbook in a graduate course on theory and/or finite element analysis of polymers and polymeric composites. An introductory course on mechanics of materials as well as on the finite element method may prove to be helpful. While the authors tried to minimize any errors in the book, it is likely that they missed some. The authors request readers to send their comments and corrections to *sroy@eng.ua.edu* or *jnreddy@tamu.edu*.

The authors express their gratitude to Dr. Vinu Unnikrishnan and Mr. Priyank Upadhyaya (University of Alabama) for their invaluable help in the preparation of the manuscript. The authors also express their sincere thanks to Mr. Jonathan Plant, Senior Editor (Engineering) at CRC Press, for his support in producing this book.

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Samit Roy is the William D. Jordan Chair Professor of Aerospace Engineering and Mechanics at University of Alabama, Tuscaloosa. Before moving to an academic position, he was a Senior Research Engineer at the Southwest Research Institute (SWRI), San Antonio, Texas. Dr. Roy has authored 60 journal papers, 11 book chapters, and more than 80 conference papers. Professor Roy's research interest is directed toward multiscale modeling and failure prediction of fiber reinforced polymer composites and structural adhesives subjected to environmental conditions, using the finite element method. His research centers around the development of mechanism-based multiscale structural durability models that would accurately predict long-term performance of materials based on data from accelerated short-term tests. He is also actively involved in the application and simulation of nanostructured reinforcements in enhancing performance of composite materials. At SWRI, he has worked on numerous research projects, including several NASA and Air Force funded projects on the use of polymer matrix composites (PMC) for cryogenic storage as well as for high-temperature supersonic airframe. He was an invited panelist at the Workshop on Composites for Extremely Cold Temperatures and Extraterrestrial Applications, Fairbanks, Alaska, organized by National Science Foundation in August 2004. He was a plenary speaker at the Composites Durability Conference organized jointly by NSF-Cambridge University in September 2007. He was the recipient of Outstanding Teaching Awards and Faculty Excellence Awards at UMR in 1999 and 2000. Dr. Roy is a Fellow of the American Society of Mechanical Engineers and an Associate Fellow of the American Institute of Aeronautics and Astronautics (AIAA), and he was the elected chairman of the AIAA Materials Technical Committee in 2009. He is a member of the editorial boards of *Polymer and Polymer Composites* and *Mechanics of Advanced Materials and Structures* journals. A more complete resume with links to journal papers can be found at **http://unix.eng.ua.edu/∼sroy**.

J. N. Reddy is a University Distinguished Professor, Regents Professor, and the Holder of Oscar S. Wyatt Endowed Chair at Texas A&M University, College Station. He is the author of more than 485 journal papers and 18 books on composite materials, plates and shells, and the finite element method. Dr. Reddy is recognized by **ISI Highly Cited Researchers** with more than 10,000 citations and h-index greater than 50 (the h-index as per Google Scholar is 60). Professor Reddy is the recipient of numerous awards including the Walter L. Huber Civil Engineering Research Prize of the American Society of Civil Engineers (ASCE), the Worcester Reed Warner Medal and the Charles Russ Richards Memorial Award of the American Society of Mechanical Engineers (ASME), the 1997 Archie Higdon Distinguished Educator Award from the American Society of Engineering Education, the 1998 Nathan M. Newmark Medal from the American Society of Civil Engineers, the 2000 Excellence in the Field of Composites from the American Society of Composites, and the 2003 Computational Solid Mechanics Award from the U.S. Association of Computational Mechanics. Dr. Reddy received honorary degrees (Honoris Causa) from the Technical University of Lisbon, Portugal, in 2009 and Odlar Yurdu University, Azerbaijan, in 2011. Professor Reddy is a Fellow of AIAA, ASCE, ASME, the American Academy of Mechanics, the American Society of Composites, the U.S. Association of Computational Mechanics, the International Association of Computational Mechanics, and the Aeronautical Society of India. Professor Reddy is the Editor-in-Chief of *Mechanics of Advanced Materials and Structures*, *International Journal of Computational Methods in Engineering Science and Mechanics*, and *International Journal of Structural Stability and Dynamics*; he also serves on the editorial boards of more than two dozen other journals. A more complete resume with links to journal papers can be found at **http://isihighlycited.com/** or **http://www.tamu.edu/acml**.