Energy Principles and Variational Methods in Applied Mechanics

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John Wiley & Sons, 2002

To

The Red, White, and Blue
(The United States of America)
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with sincere gratitude and appreciation

Preface

The increasing use of numerical and computational methods in engineering and applied sciences has shed new light on the importance of energy principles and variational methods. The number of engineering courses that make use of energy principles and variational formulations and methods has also grown very rapidly in recent years. In view of the increase in the use of the variational formulations and methods (including the finite element method), there is a need to introduce the concepts of energy principles and variational methods and their use in the formulation and solution of problems of mechanics to both undergraduate and beginning graduate students. This book, an extensively revised version of the author's earlier book *Energy and Variational Methods in Applied Mechanics*, is intended for senior undergraduate students and beginning graduate students in aerospace, civil, and mechanical engineering, and applied mechanics, who have had a course in fundamental engineering subjects as well as in ordinary and partial differential equations.

The book is organized into 10 chapters and is self-contained as far as the subject matter is concerned. Chapter 1 presents a general introduction to the subject of variational principles. Chapter 2 contains a brief review of the algebra and calculus of vectors and Cartesian tensors. A review of the basic equations of linear solid continuum mechanics is included in Chapter 3. These equations are frequently referred in subsequent chapters. Much of the material presented in Chapters 1 through 3 can be assigned as a reading material, especially in a graduate class.

Chapter 4 deals with the concepts of work, energy, and the basic topics from variational calculus, including the Euler equations, fundamental lemma of calculus of variations, essential and natural boundary conditions, and minimization of functionals without and with equality constraints. Virtual work and energy principles, and energy methods of solid and structural mechanics are presented in Chapter 5. Chapter 6 is devoted to a discussion of Hamilton's principle for dynamical systems. Classical variational methods of approximation (e.g., the methods of Ritz, Galerkin, Kantorovich, etc.) are presented in Chapter 7. All of the concepts and methods presented in Chapters 4 through 7 are illustrated using bars and beams although the methods discussed in Chapter 7 are readily applicable to field problems whose differential equations resemble those of bars and beams. Chapter 8 is dedicated to applications of the energy principles and variational methods developed in earlier chapters to circular and rectangular plates. In the interest of completeness and for use as a reference for approximate solutions, exact solutions are also included. The finite element method is introduced in Chapter 9, with applications to beams and plates. Displacement finite element models of Euler–Bernoulli and Timoshenko beam theories and classical and first-order shear deformation plate theories are presented. A unified approach, more general than that found in most solid mechanics books, is used to introduce the finite element method. As a result, the student can readily extend the method to other subject areas of solid mechanics as well as to other branches of engineering. Lastly, the mixed variational principles of Hellinger and Reissner for elasticity are derived in Chapter 10. Mixed variational formulations, including mixed finite element models of beams and plates are discussed.

Each chapter of the book contains many example problems and exercises that illustrate, test, and broaden the understanding of the topics covered. A list of references, by no means complete or up-to-date, is also provided at the end of each chapter. Answers to selective problems are included at the end of the book.

The book is suitable as a textbook for a senior undergraduate course or a first-year graduate course on energy principles and variational methods taught in aerospace, civil, and mechanical engineering, and applied mechanics departments. To gain the most from the text the student should have a senior undergraduate or first year graduate standing in engineering. Some familiarity with basic courses in differential equations, mechanics of materials, and dynamics would also be helpful.

I have benefited by the professional works, encouragement, and support of many colleagues as well as students who have taught me how to explain complicated concepts in simple terms. While it is not possible to name all of them, without their help and support it would not have been possible for me to make the modest contributions to the field of mechanics through teaching, research, and writings.

My sincere thanks are due to my teacher Professor J. T. Oden (University of Texas at Austin), for many things I learned from him and have been useful all my life. In the same vein I wish to thank Professor C. W. Bert (University of Oklahoma, Norman) for giving me the opportunity to teach and develop into what I am. I am very grateful for their mentorship, advice, and support.

Deep gratitude to my wife for her patience and support while I am occupied with the writing of this book and others, and for her smile when I say every day 'I have so much to do.'

Most books are not free of errors, especially those with many mathematical equations and numbers. I wish to thank in advance those readers who are willing to draw attention to typos and errors, using the e-mail address: *jnreddy@hotmail.com*.

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