Mechanics of Laminated Composite Plates and Shells

Theory and Analysis

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Second Edition

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To the Memory of

My parents,
My brother,
My brother in-law,
My father in-law,
Hans Eggers,
Kalpana Chawla, ...

About the Author

J. N. Reddy is a Distinguished Professor and the inaugural holder of the Oscar S. Wyatt Endowed Chair in the Department of Mechanical Engineering at Texas A&M University, College Station, Texas. Prior to his current position, he worked as a postdoctoral fellow at the University of Texas at Austin (1973–74), as a research scientist for Lockheed Missiles and Space Company (l974), and taught at the University of Oklahoma (1975–1980) and Virginia Polytechnic Institute and State University (1980–1992), where he was the inaugural holder of the Clifton C. Garvin Endowed Professorship.

Professor Reddy is the author of over 300 journal papers and 13 text books on theoretical formulations and finite-element analysis of problems in solid and structural mechanics (plates and shells), composite materials, computational fluid dynamics and heat transfer, and applied mathematics. His contributions to mechanics of composite materials and structures are well known through his research on refined plate and shell theories and their finite element models.

Professor Reddy is the first recipient of the University of Oklahoma College of Engineering's Award for Outstanding Faculty Achievement in Research, the 1984 *Walter L. Huber Civil Engineering Research Prize* of the American Society of Civil Engineers (ASCE), the 1985 Alumni Research Award at Virginia Polytechnic Institute, and 1992 *Worcester Reed Warner Medal* and 1995 *Charles Russ Richards Memorial Award* of the American Society of Mechanical Engineers (ASME). He received German Academic Exchange (DAAD) and von Humboldt Foundation (Germany) research awards. Recently, he received the 1997 *Melvin R. Lohmann Medal* from Oklahoma State University's College of Engineering, Architecture and Technology, the 1997 *Archie Higdon Distinguished Educator Award* from the Mechanics Division of the American Society of Engineering Education, the 1998 *Nathan M. Newmark Medal* from the American Society of Civil Engineers, the 2000 *Excellence in the Field of Composites Award* from the American Society of Composite Materials, the 2000 *Faculty Distinguished Achievement Award for Research*, the 2003 *Bush Excellence Award for Faculty in International Research* award from Texas A&M University, and 2003 *Computational Structural Mechanics Award* from the U.S. Association for Computational Mechanics.

Professor Reddy is a fellow of the American Academy of Mechanics (AAM), the American Society of Civil Engineers (ASCE), the American Society of Mechanical Engineers (ASME), the American Society of Composites (ASC), International Association of Computational Mechanics (IACM), U.S. Association of Computational Mechanics (USACM), the Aeronautical Society of India (ASI), and the American Society of Composite Materials. Dr. Reddy is the Editor-in-Chief of the journals *Mechanics of Advanced Materials and Structures* (Taylor and Francis), *International Journal of Computational Engineering Science* and *International Journal Structural Stability and Dynamics* (both from World Scientific), and he serves on the editorial boards of over two dozen other journals.

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Preface to the Second Edition

In the seven years since the first edition of this book appeared some significant developments have taken place in the area of materials modeling in general and in composite materials and structures in particular. Foremost among these developments have been the smart materials and structures, functionally graded materials (FGMs), and nanoscience and technology – each topic deserves to be treated in a separate monograph. While the author's expertise and contributions in these areas are limited, it is felt that the reader should be made aware of the developments in the analysis of smart and FGM structures. The subject of nanoscience and technology, of course, is outside the scope of the present study. Also, the first edition of this book did not contain any material on the theory and analysis of laminated shells. It should be an integral part of any study on laminated composite structures.

The focus for the present edition of this book remains the same – the education of the individual who is interested in gaining a good understanding of the mechanics theories and associated finite element models of laminated composite structures. Very little material has been deleted. New material has been added in most chapters along with some rearrangement of topics to improve the clarity of the overall presentation. In particular, the material from the first three chapters is condensed into a single chapter (Chapter 1) in this second edition to make room for the new material. Thus Chapter 1 contains certain mathematical preliminaries, a study of the equations of anisotropic elasticity, and an introduction to the principle of virtual displacements and classical variational methods (the Ritz and Galerkin methods). Chapters 2 through 7 correspond to Chapters 4 through 9, respectively, from the first edition, and they have been revised to include smart structures and functionally graded materials. A completely new chapter, Chapter 8, on theory and analysis of laminated shells is added to overcome the glaring omission in the first edition of this book. Chapters 9 and 10 (corresponding to Chapters 10 and 13 in the first edition) are devoted to linear and nonlinear finite element analysis, respectively, of laminated plates and shells. These chapters are extensively revised to include more details on the derivation of tangent stiffness matrices and finite element models of shells with numerical examples. Chapters 11 and 12 in the present edition correspond to Chapters 11 and 12 of the first edition, which underwent significant revisions to include laminated shells. The problem sets essentially remained the same with the addition of a few problems here and there.

The acknowledgments and sincere thanks and feelings expressed in the preface to the first edition still hold but they are not repeated here. It is a pleasure to acknowledge the help of my colleagues, especially Dr. Zhen-Qiang Cheng, for their help with the proofreading of the manuscript. Thanks are also due to Mr. Román Arciniega for providing the numerical results of some examples on shells included in Chapter 9.

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Preface to the First Edition

The dramatic increase in the use of composite materials in all types of engineering structures (e.g., aerospace, automotive, and underwater structures, as well as in medical prosthetic devices, electronic circuit boards, and sports equipment) and the number of journals and research papers published in the last two decades attest to the fact that there has been a major effort to develop composite material systems, and to analyze and design structural components made from composite materials.

The subject of composite materials is truly an interdisciplinary area where chemists, material scientists, chemical engineers, mechanical engineers, and structural engineers contribute to the overall product. The number of students taking courses in composite materials and structures has steadily increased in recent years, and the students are drawn to these courses from a variety of disciplines. The courses offered at universities and the books published on composite materials are of three types: material science, mechanics, and design. The present book belongs to the *mechanics* category.

The motivation for the present book has come from many years of the author's research and teaching in laminated composite structures and from the fact there does not exist a book that contains a detailed coverage of various laminate theories, analytical solutions, and finite element models. The book is largely based on the author's original work on refined theories of laminated composite plates and shells, and analytical and finite element solutions he and his collaborators have developed over the last two decades.

Some mathematical preliminaries, equations of anisotropic elasticity, and virtual work principles and variational methods are reviewed in Chapters 1 through 3. A reader who has had a course in elasticity or energy and variational principles of mechanics may skip these chapters and go directly to Chapter 4, where certain terminology common to composite materials is introduced, followed by a discussion of the constitutive equations of a lamina and transformation of stresses and strains. Readers who have had a basic course in composites may skip Chapter 4 also.

The major journey of the book begins with Chapter 5, where a complete derivation of the equations of motion of the classical and first-order shear deformation laminated plate theories is presented, and laminate stiffness characteristics of selected laminates are discussed. Chapter 6 includes applications of the classical and first-order shear deformation theories to laminated beams and plate strips in cylindrical bending. Here analytical solutions are developed for bending, buckling, natural vibration, and transient response of simple beam and plate structures. Chapter 7 deals with the analysis of specially orthotropic rectangular laminates using the classical laminated plate theory (CLPT). Here, the parametric effects of material anisotropy, lamination scheme, and plate aspect ratio on bending deflections and stresses, buckling loads, vibration frequencies, and transient response are discussed.

Analytical solutions for bending, buckling, natural vibration, and transient response of rectangular laminates based on the Navier and Lévy solution approaches are presented in Chapters 8 and 9 for the classical and first-order shear deformation plate theories (FSDT), respectively. The Rayleigh-Ritz solutions are also discussed for laminates that do not admit the Navier solutions. Chapter 10 deals with finite element analysis of composite laminates. One-dimensional (for beams and plate strips) as well as two-dimensional (plates) finite element models based on CLPT and FSDT are discussed and numerical examples are presented.

Chapters 11 and 12 are devoted to higher-order (third-order) laminate theories and layerwise theories, respectively. Analytical as well as finite element models are discussed. The material included in these chapters is up to date at the time of this writing. Finally, Chapter 13 is concerned about the geometrically nonlinear analysis of composite laminates. Displacement finite element models of laminated plates with the von Kármán nonlinearity are derived, and numerical results are presented for some typical problems.

The book is suitable as a reference for engineers and scientists working in industry and academia, and it can be used as a textbook in a graduate course on theory and/or analysis of composite laminates. It can also be used for a course on stress analysis of laminated composite plates. An introductory course on mechanics of composite materials may prove to be helpful but not necessary because a review of the basics is included in the first four chapters of this book. The first course may cover Chapters 1 through 8 or 9, and a second course may cover Chapters 8 through 13.

The author wishes to thank all his former doctoral students for their research collaboration on the subject. In particular, Chapters 7 through 13 contain results of the research conducted by Drs. Ahmed Khdeir, Stephen Engelstad, Asghar Nosier, and Donald Robbins, Jr. on the development of theories, analytical solutions, and finite element analysis of equivalent single-layer and layerwise theories of composite laminates. The research of the author in composite materials was influenced by many researchers. The author wishes to thank Professor Charles W. Bert of the University of Oklahoma, Professor Robert M. Jones of the Virginia Polytechnic Institute and State University, Professor A. V. Krishna Murty of the Indian Institute of Science, and Dr. Nicholas J. Pagano of Wright-Patterson Air Force Base. It is also the author's pleasure to acknowledge the help of Mr. Praveen Grama, Mr. Dakshina Moorthy, and Mr. Govind Rengarajan for their help with the proofreading of the manuscript. The author is indebted to Dr. Filis Kokkinos for his dedication and innovative and creative production of the final artwork in this book. Indeed, without his imagination and hundreds of hours of effort the artwork would not have looked as beautiful, professional, and technical as it does.

The author gratefully acknowledges the support of his research in composite materials in the last two decades by the Office of Naval Research (ONR), the Air Force Office of Scientific Research (AFOSR), the U.S. Army Research Office (ARO), the National Aeronautics and Space Administration (NASA Lewis and NASA Langley), the U.S. National Science Foundation (NSF), and the *Oscar S. Wyatt Chair* in the Department of Mechanical Engineering at Texas A&M University. Without this support, it would not have been possible to contribute to the subject of this book. The author is also grateful to Professor G. P. Peterson, a colleague

and friend, for his encouragement and support of the author's professional activities at Texas A&M University.

The writing of this book took thousands of hours over the last ten years. Most of these hours came from evenings and holidays that could have been devoted to family matters. While no words of gratitude can replace the time lost with family, it should be recorded that the author is grateful to his wife Aruna for her care, devotion, and love, and to his daughter Anita and son Anil for their understanding and support.

During the long period of writing this book, the author has lost his father, brother, brother in-law, father in-law, and a friend (Hans Eggers) - all suddenly. While death is imminent, the suddenness makes it more difficult to accept. This book is dedicated to the memory of these individuals.

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All that is not given is lost