

# FINITE ELEMENT ANALYSIS OF COMPOSITE LAMINATES

# SOLID MECHANICS AND ITS APPLICATIONS

## Volume 7

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# Finite Element Analysis of Composite Laminates

by

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# *Preface*

Composite materials are increasingly used in aerospace, under water, and automotive structures. The application of composite materials to engineering components has spurred a major effort to analyze structural components made from them. Composite materials provide unique advantages over their metallic counterparts, but they also present complex and challenging problems to analysts and designers. To take advantage of the full potential of composite materials, structural analysts and designers must have accurate mathematical models and design methods at their disposal. The most common structural elements are plates and shells. An accurate modelling of stress fields and failures is of paramount importance in the design of such components.

The present monograph has the objective of introducing the mechanics concepts, structural theories, and finite element models of composite laminates. Detailed coverage of the basic mechanics of composite materials, theories of composite plates and shells, and the finite element method are avoided in the interest of providing a general background necessary for engineers to analyze composite structures.

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