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CON APLICACIONES A INGENIERÍA Y CIENCIAS

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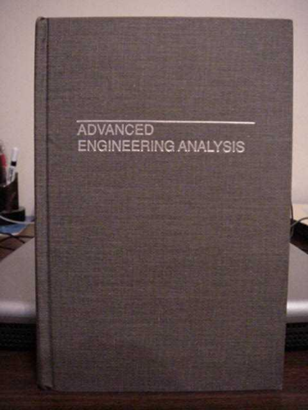
ADVANCED
ENGINEERING ANALYSIS

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PREFACE

The objective of this book is to present for engineers and applied scientists the basic mathematical concepts of vector and tensor analysis, the extension of these concepts into abstract function spaces (functional analysis), and the unification of these subjects with the variational calculus and associated methods of numerical approximation. Vector and tensor analysis is fundamental to understanding and dealing with a vast range of physical problems and disciplines, and is an indispensable tool for engineering analysis as a subject in itself. In addition, the classical notion of vectors and tensors in Euclidean space, with its physical applications, leads naturally to the modern abstract notion of vectors in function spaces, and thus to the subject of functional analysis. These abstract notions of vector and function spaces provide powerful new concepts and tools of analysis. In particular, they lend themselves directly to approximation methods stemming from the calculus of variations. The variational calculus in turn is related intimately to vector analysis in its complementary representation and interpretation of physical phenomena. Thus the three subjects of this book, vector and tensor analysis, functional analysis, and variational calculus, are mutually related and form a fundamental foundation for modern engineering analysis.

This book is the outgrowth of class notes which the authors have developed and taught over a decade at four major universities. The book is intended for undergraduate seniors and first-year graduate students in engineering and the applied sciences. Senior standing in engineering or a course in differential equations is a prerequisite for the understanding of the material in this book. The subject matter should serve as text for a two quarter course, or a one-semester course in any two of the three chapters on engineering analysis.

The text is divided into three parts: 1. Elements of Vector and Tensor Analysis, 2. Elements of Functional Analysis, and 3. Calculus of Variations and Variational Methods. Numerous examples, most of which are applications of the concepts to problems in various fields of engineering, are provided throughout the book. Many exercise problems are included to test and extend the understanding of the subject matter. A number of these exercise problems are intended to explore related ideas and applications of the concepts covered.

The conclusions of proofs and examples are indicated by the symbol ■.

There are several sections that can be skipped in a first reading of the book (or, if required as prerequisite material, omitted in the syllabus of the course). The material is intended for a semester or two quarter courses, although the material is better suited for a two quarter sequence (Elements of Vectors and Tensors at the undergraduate senior level and Elements of Functional Analysis and Calculus of Variations and Variational Methods at the first-year graduate level).

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J. N. REDDY
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