

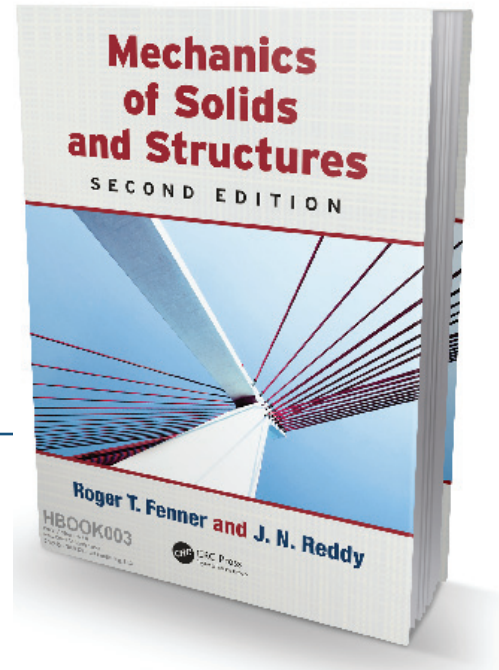


Mechanics of Solids and Structures, Second Edition

A revision of a popular textbook, this volume emphasizes the development of analysis techniques from basic principles for a broad range of practical problems, including simple structures, pressure vessels, beams and shafts. The book integrates numerical and computer techniques with programs for carrying out analyses, facilitating design, and solving the problems found at the end of each chapter. It also presents the underlying theory and traditional manual solution methods along with these techniques. This new second edition covers relationships between stress and strain, torsion, statically determinate systems, instability of struts and columns, and compatibility equations.

Key Features

- Includes color graphics and illustrations throughout for better visual understanding of mechanics
- Presents balanced coverage of traditional mechanics and modern numerical/computer methods, including finite element analysis
- Includes a new chapter on energy methods as applied to engineering mechanics
- Prepares students for graduate-level work in engineering mechanics
- Includes numerous engineering examples, applications and problems



Roger T. Fenner
J.N. Reddy

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Introduction. Statically Determinate Systems. Relationships between Stress and Strain. Statically Indeterminate Systems. Bending of Beams: Moments, Forces, and Stresses. Bending of Beams: Deflections. Torsion. Instability and the Buckling of Struts and Columns. Transformations of Stress and Strain. Equilibrium and Compatibility Equations: Beams and Thick-Walled Cylinders. Energy Methods of Structural Mechanics. Appendices. Index.

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List of Symbols

The symbols that are used throughout the book are defined in the following list. In some cases, particular symbols have different meanings in different parts of the book, although this should not cause any serious ambiguity as will be clear in the context. Other symbols or alternative definitions of the present symbols representing, for example, dimensions of components or constants of integration are introduced within the limited contexts of particular examples or parts of analysis.

A	Area (especially cross-sectional area)
\mathbf{A}	Coefficient matrix
b	Breadth
C	Couple
C_1 to C_5	Constants in a finite element interpolation function
c_1, c_2	Distances from neutral surface to highest and lowest points of a beam cross section
D	Diameter
d	Depth
E	Young’s modulus (modulus of elasticity)
e	Normal strain
e	Eccentricity
e_{vol}	Volumetric strain
e_Y	Yield strain
F	Force
\mathbf{F}	Vector of applied external loads
f_r	Body force per unit volume in the r direction
f_x, f_y	Body forces per unit volume in the x and y directions
f	Load per unit length of a beam or cable
$f(\)$	Function; step or singularity function
\mathbf{f}	Element load vector
G	Shear modulus (modulus of rigidity)
g	Acceleration due to gravity
H	Horizontal component of force
h	Height
I	Second moment of area of a beam cross section about its neutral axis
i, j	Node numbers
J	Polar second moment of area of a shaft cross section about its axis

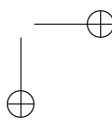
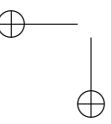
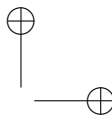
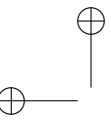
K	Stress concentration factor; bulk modulus; radius ratio for a thick-walled cylinder
\mathbf{K}	Global stiffness matrix
k	Stiffness
$\mathbf{K}^{(m)}$	Element stiffness matrix of element m
L	Length
L_e	Effective length of the equivalent pin-ended strut
M	Moment; bending moment
m	Element number; modulus ratio
N	Number (of elements, reactions, supports, etc., indicated by appropriate subscripts); moment
n	Buckling parameter defined in equation (8.5)
P	Force
P_c	Euler critical buckling force
p	Pressure; perimeter
q	Distributed transverse load (beams)
q_0	Intensity of the distributed transverse load, q (beams)
\mathbf{q}	Vector of applied external loads
Q	Force; first moment of area of the region of a beam cross section above a given distance from the neutral axis
R	Reaction force at a support; radius; radius of curvature of the neutral surface of a beam
r	Radial coordinate in the cylindrical polar system; radius of gyration
S	Elastic section modulus
T	Force in a cable or member of a structure; temperature; torque
\mathbf{T}	Vector of member forces
t	Wall thickness
U, V	Forces in the x and y directions
u, v, w	Displacements in the $x, y,$ and z directions
V	Vertical component of force; shear force; volume
\mathbf{V}	Vector of forces and moments
w	Weight per unit length (cables)
W	Total weight
X, Y	Global Cartesian coordinates
x, y, z	Cartesian coordinates
z	Axial coordinate in the cylindrical polar system
α	Angle; coefficient of linear thermal expansion
β	Coefficient of volumetric thermal expansion
γ	Shear strain
Δ	Change of (followed by another symbol)
$\mathbf{\Delta}$	Vector of displacements

δ	Displacement; radial interference
δ	Variational operator used in Chapter 11
ε	Natural strain
θ	Angular coordinate in the cylindrical polar system; angle
λ	Extension ratio; rotational stiffness
ν	Poisson’s ratio
ρ	Density
σ	Normal stress
σ_e	Von Mises equivalent stress
σ_H	Hydrostatic stress
σ_U	Ultimate tensile stress
σ_Y	Yield stress
τ	Shear stress
ϕ	Angle
ω	Angular velocity

List of Computer Programs

(Available at <http://www.crcpress.com/product/isbn/9781439858141>)

-
-
- **SDPINJ** (Analysis of Statically Determinate PIN-Jointed structures)
 - **SIPINJ** (Analysis of Statically Indeterminate PIN-Jointed structures)
 - **SDBEAM** (Analysis of Statically Determinate BEAMs)
 - **SIBEAM** (Analysis of Statically Indeterminate BEAMs)
 - **ROSETTE** (Determination of states of strain and stress from strain rosette measurements)
 - **ANAL2D** (Analysis of two-dimensional states of stress or strain at a point)
 - **CYLIND** (Analysis of stresses and strains in two compounded thick-walled cylinders subject to internal pressure)
 - **SOLVE** (Subroutine for the solution of linear algebraic equations by Gaussian elimination)
 - **ROOT** (Solution of a nonlinear algebraic equation by the bisection method)



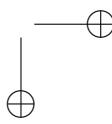
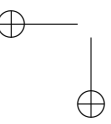
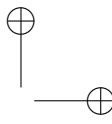
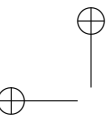
Preface (to the second edition)

In the more than two decades since the first edition of this book appeared, there have been significant developments in the computational methods used for the solution of problems in solid and structural mechanics. The advances made in the performance of desktop computing platforms have further revolutionized the way in which engineering problems are solved in practice. However, the basic principles to be mastered in the study of the subject have not changed. Consequently, all of the contents of the first edition remain intact except for minor changes here and there and the addition of a chapter, Chapter 11, on energy methods of structural mechanics. This chapter may be considered a bit advanced, but once the reader understands how to construct the expressions for total potential energy and complementary potential energy associated with the structure, the principles of minimum total potential energy and complementary potential energy provide the means to obtain governing equations of the structure, while Castigliano’s Theorems I and II provide a means to determine point forces and displacements with ease. The material presented in this chapter will also aid readers in gaining further understanding of the finite element method, which is the most popular computational method ever devised to solve structural mechanics problems of practical importance. Another change to the first edition is that all of the listings of the computer programs discussed in various chapters and Appendix A and Appendix D from the first edition are removed, and are now available on the publisher’s website for the book.

As with the original text, the objective of the present book is to serve as the first course in the mechanics of solids and structures (sometimes referred to as *Strength of Materials* or *Mechanics of Deformable Solids*) offered in aerospace, civil, and mechanical engineering departments. The distinctive feature of the original and current edition over its competitors is the integration of a number of computer techniques and programs into the body of the text for carrying out structural analysis. The Preface to the first edition provides the background, motivation, and approach used and they are not repeated here. Additional material is available from the CRC website: <http://www.crcpress.com/product/isbn/9781439858141>

The authors are grateful to Jonathan Plant, Executive Editor at Taylor & Francis/CRC Press, for bringing this important text back to life. The second author is grateful to Professor Roger Fenner as well as to Jonathan Plant for allowing him to undertake the task of revising and adding material to the first edition. The second author also gratefully acknowledges the invaluable help of Dr. Vinu Unnikrishnan (now at the University of Alabama, Tuscaloosa), with the LaTeX commands to produce the manuscript of the second edition.

Roger T. Fenner and J. N. Reddy



Preface (to the first edition)

There are only a few basic principles to be mastered in the study of the subject variously referred to as the mechanics of solids, mechanics of materials or strength of materials. It is the application of these principles to the solution of problems, and the choice of assumptions which must be made, which present the greatest challenge. This text is intended for the first course in mechanics of solids offered to engineering students. It concentrates on developing analysis techniques from basic principles for a range of practical problems, which includes simple structures, pressure vessels, beams and shafts. Many worked examples are given. The arrival of computers in general, and personal computers in particular, has revolutionized the way in which engineering problems are solved in practice, and this is being reflected in the way in which subjects such as the mechanics of solids are taught. A distinctive feature of the present book is therefore the inclusion of a number of computer techniques and programs for carrying out the analyses – not merely as appendices, but integrated into the text. The programs will also find many applications in the teaching of design.

It is not intended that the use of computer programs should replace hand calculations in the learning process, but should supplement them, and make it possible for the student to explore more complex and realistic problems of analysis and design. The approach adopted is therefore first to present the underlying theory and the traditional manual methods of solution before introducing computer techniques. The programs, which are coded in FORTRAN 77, are suitable for personal computers, but can also be run on minicomputers or mainframes. Detailed internal and external documentation is provided to aid the understanding of the programs, together with examples of their use. It is intended that students should use them to solve many of the problems which are set at the end of each chapter, and in their design work.

It is assumed that students using this book will have some experience of elementary statics (mechanics of rigid bodies), although Chapter 1 includes a review of the relevant topics. A knowledge of matrix notation for the presentation of linear algebraic equations is also assumed. Some familiarity with the solution of constant coefficient ordinary differential equations (particularly for the buckling problems considered in Chapter 8) is highly desirable, as is experience of partial differentiation and integration if Chapter 10 on more advanced applications is to be studied. Those numerical analysis techniques which are incorporated in the computer programs, particularly for the solution of simultaneous linear algebraic equations and single nonlinear algebraic equations, are described in appendices. It is assumed that students will have sufficient knowledge of the FORTRAN programming language to be able to read and understand relatively straightforward programs.

There are many ways in which a first course on the mechanics of solids can be presented. The approach adopted here, based on the author's experience of teaching the subject, is to start with types of problems involving uniform stresses. Initially such stresses are uniaxial, as in pin-jointed structures, progressing to biaxial and even triaxial, but without shearing, as in thin-walled pressure vessels. Statically determinate situations, which require only the consideration of equilibrium conditions for the forces and stresses to be found, are treated before statically indeterminate ones. Problems involving relatively simple variations of stresses are then examined, principally the bending of beams and the torsion of shafts. Finally, an introduction to more complex situations is provided via the analysis of two-dimensional states of stress and strain, failure criteria and the differential equations of equilibrium and compatibility in two dimensions.

In addition to a review of statics, Chapter 1 introduces the concepts of stress and strain in a solid body, the influence of material properties and the principles of the mechanics of solids. These principles are those of equilibrium of forces, compatibility of strains and the stress-strain characteristics of materials, underlying themes which run through the remainder of the book. In Chapter 2, some statically determinate systems are analyzed, in particular pin-jointed structures, thin cylindrical and spherical shells, and flexible cables. A computer program is introduced for the analysis of statically determinate pin-jointed structures, using a simple form of the finite element method. Stress-strain relationships for engineering materials are discussed in Chapter 3 and are used to find the deformations of statically determinate systems considered in the previous chapter. Some types of statically indeterminate systems are examined in Chapter 4, notably pin-jointed structures (by the finite element method), liquid-filled pressure vessels and problems involving resisted thermal expansion.

Chapters 5 and 6, which form a major part of the book, are concerned with beams and the simple theory of bending. While Chapter 5 deals with shear forces, bending moments and stresses, and the analysis of statically determinate beams (including a finite element analysis computer program), Chapter 6 is concerned with beam deflections, leading to the analysis of statically indeterminate beams (and another finite element analysis program). In Chapter 7, problems of torsion of circular shafts are considered. Following an introduction to problems of instability, Chapter 8 deals with the buckling of struts and columns.

In Chapter 9, attention moves away from problems involving only simple states of mainly uniaxial stresses toward more complex situations. Transformations of stress and strain components acting on different planes at a point lead to the definition of principal and maximum shear values. A computer method is introduced for analyzing stresses or strains at a point, as is a program for determining the state of strain and stress at a point from strain gage measurements. Criteria for yielding and fracture

under complex states of stress are then examined. Finally, Chapter 10 develops the principles of equilibrium and compatibility into the partial differential stress equilibrium and strain compatibility equations for problems involving general one- and two-dimensional variations of stresses and strains. These are applied to beam problems and serve to demonstrate the levels of approximation involved in the simple theory of bending. In a one-dimensional form suitable for axisymmetric problems, they are also applied to rotating disks and thick-walled cylinders used as pressure vessels, and a computer method is introduced for the determination of stresses and strains in compound thick-walled cylinders.

The coverage of topics provided by the text may well be greater than that required for particular courses. For example, not all instructors would wish to deal so fully with pin-jointed structures, although the solution of more realistic problems is so much more practical using computer techniques, and they do provide a natural introduction to the finite element method, which students will meet in later courses. Also, Chapter 10, and perhaps parts of Chapter 9, may be more appropriate for a more advanced course. The introduction of computer techniques has meant that some more traditional methods have been omitted. For example, graphical methods for the analysis of pin-jointed structures are not considered. Similarly, the only manual method described for the determination of beam deflections involves integration of the moment–curvature equation: the more graphical moment–area method is not covered. Other topics which have been omitted, but which are only rarely covered to any significant depth in a first course, are energy methods and an introduction to plasticity and the analysis of elastic-plastic bending and torsion problems.

Some of the computer techniques are described in the text as the finite element technique. This is because they are the kinds of essentially simple techniques which led to the birth of the finite element method in structural mechanics. It is useful to describe them in terms of the finite element method, not least because it provides a good preparation for the introduction of more sophisticated finite element approaches in later courses.

Many engineering courses have now converted entirely to SI metric units. In the United States, however, US customary units are still widely used, reflecting industrial practice. Consequently, in this book both sets of units are employed. In worked examples, given numerical data and the main calculated results are usually shown first in SI units, followed in parentheses by equivalent values in US customary units. The equivalence is not intended to be exact, and values are normally quoted to only two significant figures. The intention is to provide those readers less familiar with SI units with a better feel for the magnitude of the quantities involved. In many examples, detailed calculations in both sets of units are shown, side by side. The material property data presented in Appendix A are given in both SI and US customary units, together with the appropriate conversion factors.

A substantial number of problems is provided at the end of each chapter, first a set in SI units and then a set in US customary units. Where appropriate, problems are also grouped under topic headings, and within each group they are graded in difficulty. While the elementary problems involve only straightforward application of the methods described in the text, some of the more difficult ones are more open-ended and of the design type. The necessary material properties are in most cases not given in the problems, but are provided in an appendix (Appendix A). Answers to alternate problems are listed at the end of the book, and worked solutions to all problems are contained in a separate instructor’s manual.

Magnetic disk copies of all the computer programs can be obtained from the publisher. Programs may be freely copied, used, modified or translated. Although they have been carefully tested, they may contain errors, and I would appreciate being informed of any that are found. While the programs were developed specifically for teaching purposes, they may find applications in the solution of real problems. If they are used for this purpose, I will not be responsible for any errors they may contain.

Acknowledgments

The author of a textbook owes a debt of gratitude to other people, whose experience materially contributes to its formulation. In particular I gratefully acknowledge detailed advice and comments received from the following: Professor Daniel Frederick, Department of Engineering Science and Mechanics, Virginia Polytechnic Institute and State University; Professor Carl F. Long, Thayer School of Engineering, Dartmouth College; Dr. V. J. Meyers, School of Civil Engineering, Purdue University; and Dr. Ozer A. Arnaz, Department of Mechanical Engineering, California State University, Sacramento. General guidance and support from Dr. Lee L. Lowery, Department of Civil Engineering, Texas A&M University, College Station, is also much appreciated.

I also wish to acknowledge the contributions, both direct and indirect, made by many of my colleagues at Imperial College to the development of the course with its associated examples, problems and examination questions, on which the book is based. In particular, I would like to thank Dr. Frank Ellis, Professor Alan Swanson, and Dr. Simon Walker. Special thanks are also due to Dr. Fusun Nadiri for her careful reading of much of the typescript and for many helpful suggestions. Some of the examples and problems used are taken from, or based on, questions set in University of London examination papers. Permission to use this material is gratefully acknowledged. Responsibility for the solutions and answers is, however, mine alone.

Roger T. Fenner