# PRINCIPLES OF CONTINUUM MECHANICS

*A Study of Conservation Principles with Applications*

**by**

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*When even the brightest mind in our world has been trained up from childhood in a superstition of any kind, it will never be possible for that mind, in its maturity, to examine sincerely, dispassionately, and conscientiously any evidence or any circumstance which shall seem to cast a doubt upon the validity of that superstition.*

Mark Twain

*The fact that an opinion has been widely held is no evidence whatever that it is not utterly absurd; indeed in view of the silliness of the majority of mankind, a widespread belief is more likely to be foolish than sensible.* Bertrand Russell

*Desire for approval and recognition is a healthy motive, but the desire to be acknowledged as better, stronger, or more intelligent than a fellow being or fellow scholar easily leads to an excessively egoistic psychological adjustment, which may become injurious for the individual and for the community.*

Albert Einstein

# Preface

*You cannot teach a man anything, you can only help him find it within himself.*
Galileo Galilei

This book is a simplified version of the author's book, *An Introduction to Continuum Mechanics with Applications,* published by Cambridge University Press (New York, 2008), intended for use as an undergraduate text book. As most modern technologies are no longer discipline-specific but involve multidisciplinary approaches, undergraduate engineering students should be educated to think and work in such environments. Therefore, it is necessary to introduce the subject of *principles of mechanics* (i.e., laws of physics applied to science and engineering systems) to undergraduate students so that they have a strong background in the basic principles common to all disciplines and be able to work at the interface of science and engineering disciplines. A first course on principles of mechanics provides an introduction to the basic concepts of stress and strain and conservation principles, and prepares engineer-scientists for advanced courses in traditional as well as emerging fields such as biotechnology, nanotechnology, energy systems, and computational mechanics. Undergraduate students with such background may seek advanced degrees in traditional (e.g., aerospace, civil, electrical, mechanical, physics, applied mathematics) as well as interdisciplinary degrees programs (e.g., bioengineering, engineering physics, nanoscience and engineering, biomolecular engineering, and so on).

There are not many books on principles of mechanics that are written keeping the undergraduate engineering or science students in mind. A vast majority of books on the subject are written for graduate students of engineering and tend to be more mathematical and too advanced to be of use for third year or senior undergraduate students. This book presents the subjects of mechanics of materials, fluid mechanics, and heat transfer in unified form using the conservation principles of mechanics. It is hoped that the book, which is simple and facilitates in presenting the main concepts of the previous three courses under a unified framework.

With a brief discussion of the concept of a continuum in Chapter 1, a review of vectors and tensors is presented in Chapter 2. Since the analytical language of applied sciences and engineering is mathematics, it is necessary for all students of this course to familiarize themselves with the notation and operations of vectors, matrices, and tensors that are used in the mathematical description of physical phenomena. Readers who are familiar with the topics of this chapter may refresh or skip and go to the next chapter. The subject of kinematics, which deals with geometric changes without regard to the forces causing the deformation, is discussed in Chapter 3. Measures of engineering normal and shear strains and definitions of mathematical strains are introduced here. Both simple one-dimensional systems to as well as two-dimensional continua are used to illustrate the strain and strain-rate measures introduced. In Chapter 4, the concept of stress vector and stress tensor are introduced. It is here, the readers are presented with entities that require two directions - namely, the plane on which they are measured and the direction in which they act - to specify them. Transformation equations among components of stress tensor referred to two different orthogonal coordinate systems are derived, and principal values and principal planes (i.e., eigenvalue problems associated with the stress tensor) are also discussed.

Chapter 5 is dedicated to the derivation of the governing equations of mechanics using the conservation principles of continuum mechanics (or laws of physics). The principles of conservation of mass, linear momentum, angular momentum, and energy are presented using one-dimensional systems as well as general three-dimensional systems. The derivations are presented in invariant (i.e., independent of a coordinate system) as well as in component form. The equations resulting from these principles are those governing stress and deformation of solid bodies, stress and rate of deformation of fluid elements, and transfer of heat through solid media. Thus, this chapter forms the heart of the course. Constitutive relations that connect the kinematic variables (e.g., density, temperature, deformation) to the kinetic variables (e.g., internal energy, heat flux, and stresses) are discussed in Chapter 6 for elastic materials, viscous fluids, and heat transfer in solids.

Chapter 7 is devoted to the application of the field equations derived in Chapter 5 and constitutive models presented in Chapter 6 to problems of heat conduction in solids, fluid mechanics (inviscid flows as well as viscous incompressible flows), diffusion, and solid mechanics (e.g., bars, beams, and plane elasticity). Simple boundary-value problems are formulated and their solutions are discussed. The material presented in this chapter illustrates how physical problems are analytically formulated with the aid of the equations resulting from the conservation principles.

As stated previously, the present book is an undergraduate version of the author's book *An Introduction to Continuum Mechanics* (Cambridge University Press, New York, 2008). The presentation herein is limited in scope when compared to the author's graduate level textbook. The major benefit of a course based on this book is to present the governing equations of diverse physical phenomena from a unified point of view, namely, from the conservation principles (or laws of physics) so that students of applied science and engineering see the physical principles as well as the mathematical structure common to diverse fields. Readers interested in advanced topics may consult the author's continuum mechanics book cited above or other titles listed in references therein.

The author is pleased to acknowledge the fact that the manuscript was tested with the undergraduate students in the College of Engineering at Texas A&M University as well as in the Engineering Science Programme at the National University of Singapore. The students, in general, have liked the contents and the simplicity with which the concepts are introduced and explained. They also expressed the feeling that the subject is more challenging than most at the undergraduate level but a useful prerequisite to graduate courses in engineering.

The book contains so many mathematical expressions that it is hardly possible not to have typographical and other kinds of errors. The author wishes to thank in advance those who are willing to draw the author's attention to typos and errors, using the e-mail address jn_reddy@yahoo.com.

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# About the Author

**J. N. Reddy** is a Distinguished Professor and the Holder of Oscar S. Wyatt Endowed Chair in the Department of Mechanical Engineering at Texas A&M University (**http://www.tamu.edu/acml**).

Professor Reddy is a renowned researcher and educator in the broad fields of mechanics, applied mathematics, and computational engineering science. Professor Reddy's research areas include theory and finite element analysis of problems in structural mechanics (composite plates and shells), fluid dynamics, and heat transfer; theoretical modelling of stress and deformation of biological cells and soft tissues; nanocomposites; and development of robust computational technology (including the *K-version finite element models based on the least-squares method* in collaboration with Professor Karan Surana of the University of Kansas). He is the author of over 375 journal papers and 16 books on these subjects. The books published by Dr. Reddy include *Introduction to the Finite Element Method*, 3rd ed., McGraw-Hill, 2006; *Mechanics of Laminated Plates and Shells: Theory and Analysis,* 2nd ed., CRC Press, 2004; *An Introduction to Nonlinear Finite Element Analysis*, Oxford University Press, 2004; and *An Introduction to Continuum Mechanics*, Cambridge University Press, 2008.

Dr. Reddy's outstanding research credentials have earned him wide international acclaim in the form of numerous professional awards, citations, fellowship in all major professional societies including AAM, AIAA, ASC, ASCE, ASME, IACM and USACM, membership on two dozen archival journals, and numerous keynote and plenary lecture invitations at international conferences. Dr. Reddy is the Editor-in-Chief of *Applied Mechanics Reviews, Mechanics of Advanced Materials and Structures*, *International Journal of Computational Methods in Engineering Science and Mechanics*, and *International Journal of Structural Stability and Dynamics.*

The extent of Dr. Reddy's original and sustained contributions to education, research, and professional service is substantial. As a result of his extensive publications of archival journal papers and books on a wide range of topics in applied sciences and engineering, Dr. Reddy is one of the selective few researchers in engineering around world who are recognized by *ISI Highly Cited Researchers* with over 10,000 citations with H-index of over 40. In February 2009 he was awarded a *Honoris Causa* (Honorary Doctorate) by the Technical University of Lisbon.