Introduction to Continuum Mechanics

covering linearized elasticity, fluid mechanics, heat transfer, and viscoelasticity

Second Edition

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Preface to the Second Edition

Engineers are problem solvers. They construct mathematical models, develop analysis approaches and methodologies, and design and manufacture various types of systems or processes. Mathematical development and engineering analysis are aids to designing systems for specific functionality, and they involve (1) mathematical model development, (2) data acquisition by measurements, (3) numerical simulation, and (4) evaluation of the results in light of known information. Mathematical models are developed using laws of physics and assumptions concerning the system behavior. The most difficult step in arriving at a design that is both functional and cost-effective is the construct of a suitable mathematical model of the system behavior. It is in this context a course on continuum mechanics or elasticity provides engineers with a background to formulate a suitable mathematical model and evaluate it in the context of the functionality and design constraints placed on the system.

The second edition of *Introduction to Continuum Mechanics* has the same objective as the first one, namely, to facilitate an easy and thorough understanding of concepts from a first course on continuum mechanics and elasticity. The course also helps engineers who depend on canned programs to analyze problems in interpreting the results produced by such programs. **The book offers a concise but rigorous treatment of the subject of continuum mechanics and elasticity at the introductory level.** In all of the chapters of the second edition, additional explanations, examples, and problems are added. No attempt is made to enlarge the scope or increase coverage of topics.

The book may be used as a text book for a first course on continuum mechanics as well as elasticity (omitting Chapter 8 on fluid mechanics and heat transfer). A solutions manual is also prepared for the book. The solution manual is made available by the publisher only to those instructors who adopt the book as a textbook for a course. Since the publication of the first edition several users of the book communicated their compliments as well as errors they found, and the author is grateful to them. All of the errors known to the author have been corrected in the current edition. The author is grateful, in particular, to Professors Karan Surana (University of Kansas), Arun Srinivasa (Texas A&M University), Srikanth Vedantam (Indian Institute of Technology, Madras), and Rebecca Brannon (University of Utah) for their constructive comments and advice.

J. N. Reddy

Tis the good reader that makes the good book; in every book he finds passages which seem confidences or asides hidden from all else and unmistakeably meant for his ear; the profit of books is according to the sensibility of the reader; the profoundest thought or passion sleeps as in a mine, until it is discovered by an equal mind and heart. Ralph Waldo Emerson

You cannot teach a man anything, you can only help him find it within himself.
Galileo Galilei

Preface (to the First Edition)

If I have been able to see further, it was only because I stood on the shoulders of giants. Isac Newton

Many of the mathematical models of natural phenomena are based on fundamental scientific laws of physics or otherwise, extracted from centuries of research on the behavior of physical systems under the action of natural "forces". Today this subject is referred to simply as *mechanics* - a phrase that encompasses broad fields of science concerned with the behavior of fluids, solids, and complex materials. Mechanics is vitally important to virtually every area of technology and remains an intellectually rich subject taught in all major universities. It is also the focus of research in departments of aerospace, chemical, civil, and mechanical engineering, and engineering science and mechanics, as well as applied mathematics and physics. The last several decades have witnessed a great deal of research in continuum mechanics and its application to a variety of problems. As most modern technologies are no longer discipline-specific but involve multidisciplinary approaches, scientists and engineers should be trained to think and work in such environments. Therefore, it is necessary to introduce the subject of mechanics to senior undergraduate and beginning graduate students so that they have a strong background in the basic principles common to all major engineering fields. A first course on *continuum mechanics* or *elasticity* is the one that provides the basic principles of mechanics and prepares engineers and scientists for advanced courses in traditional as well as emerging fields such as biomechanics and nanomechanics.

There are many books on mechanics of continua. These books fall into two major categories: those which present the subject as highly mathematical and abstract subject, and those which are too elementary to be of use for those who will pursue further work in fluid dynamics, elasticity, plates and shells, viscoelasticity, plasticity, and interdisciplinary areas such as geomechanics, biomechanics, mechanobiology, and nanoscience. As is the case with all other books written (solely) by the author, the objective is to facilitate an easy understanding of the topics covered. While the author is fully aware that he is not an authority on the subject of this book, he feels that he understands the concepts well and feels confident to explain to others. It is hoped that the book, which is simple in presenting the main concepts yet mathematically rigorous enough in providing the invariant form as well as component form of the governing equations for analysis of practical problems of engineering. In particular, the book contains formulations and applications to specific problems from heat transfer, fluid mechanics, and solid mechanics.

The motivation and encouragement that led to the writing of this book came from the experience of teaching a course on continuum mechanics at Virginia Polytechnic Institute and State University and Texas A&M University. A course on continuum mechanics takes different forms - abstract to very applied - when taught by different people. The primary objective of the course taught by the author is two-fold: (1) formulation of equations that describe the motion and thermomechanical response of materials, and (2) solution of these equations for specific problems from elasticity, fluid flows, and heat transfer. The present book is a formal presentation of the author's notes developed for such a course over last two and half decades.

With a brief discussion of the concept of a continuum in Chapter 1, a review of vectors and tensors is presented in Chapter 2. Since the language of mechanics is mathematics, it is necessary for all readers to familiarize themselves with the notation and operations of vectors and tensors. The subject of kinematics is discussed in Chapter 3. Various measures of strain are introduced here. The deformation gradient, Cauchy–Green deformation, Green–Lagrange strain, Cauchy and Euler strain, rate of deformation, and vorticity tensors are introduced and the polar decomposition theorem is discussed in this chapter. In Chapter 4, various measures of stress - Cauchy stress and Piola–Kirchhoff stress measures - are introduced, and stress equilibrium equations are presented.

Chapter 5 is dedicated to the derivation of the field equations of continuum mechanics, which forms the heart of the book. The field equations are derived using the principles of conservation of mass, momenta, and energy. Constitutive relations that connect the kinematic variables (e.g., density, temperature, deformation) to the kinetic variables (e.g., internal energy, heat flux, and stresses) are discussed in Chapter 6 for elastic materials, viscous and viscoelastic fluids, and heat transfer.

Chapters 7 and 8 are devoted to the application of the field equations derived in Chapter 5 and constitutive models of Chapter 6 to problems of linearized elasticity, and fluid mechanics and heat transfer, respectively. Simple boundary-value problems, mostly linear, are formulated and their solutions are discussed. The material presented in these chapters illustrate how physical problems are analytically formulated with the aid of continuum equations. Chapter 9 deals with linear viscoelastic constitutive models and their application to simple problems of solid mechanics. Since a continuum mechanics course is mostly offered by solid mechanics programs, the coverage in this book is slightly more favorable, in terms of the amount and type of material covered, to solid and structural mechanics.

The book is written keeping the undergraduate seniors and first-year graduate students of engineering in mind. Therefore, it is most suitable as a text book for adoption for a first course on continuum mechanics or elasticity. The book also serves as an excellent precursor to courses on viscoelasticity, plasticity, nonlinear elasticity, and nonlinear continuum mechanics.

The book contains so many mathematical equations that it is hardly possible not to have typographical and other kinds of errors. I wish to thank in advance those readers who are willing to draw the author's attention to typos and errors, using the e-mail address: *jnreddytamu.edu*.

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About the Author

J. N. Reddy is a University Distinguished Professor, Regents Professor, and the Holder of Oscar S. Wyatt Endowed Chair in the Department of Mechanical Engineering at Texas A&M University, College Station, Texas. Prior to the current position, he was the Clifton C. Garvin Professor in the Department of Engineering Science and Mechanics at Virginia Polytechnic Institute and State University (VPI&SU), Blacksburg, Virginia.

Dr. Reddy is internationally known for his contributions to theoretical and applied mechanics and computational mechanics. He is the author of over 450 journal papers and 17 other books. Professor Reddy is the recipient of numerous awards including the *Walter L. Huber Civil Engineering Research Prize* of the American Society of Civil Engineers (ASCE), the *Worcester Reed Warner Medal* and the *Charles Russ Richards Memorial Award* of the American Society of Mechanical Engineers (ASME), the 1997 *Archie Higdon Distinguished Educator Award* from the American Society of Engineering Education (ASEE), the 1998 *Nathan M. Newmark Medal* from the American Society of Civil Engineers, the 2000 *Excellence in the Field of Composites* from the American Society of Composites, the 2003 *Bush Excellence Award for Faculty in International Research* from Texas A&M University, and the 2003 *Computational Solid Mechanics Award* from the U.S. Association of Computational Mechanics (USACM).

Professor Reddy is a *Fellow* of AIAA, ASCE, ASME, the American Academy of Mechanics (AAM), the American Society of Composites (ASC), the U.S. Association of Computational Mechanics (USACM), the International Association of Computational Mechanics (IACM), and the Aeronautical Society of India (ASI). Professor Reddy is the Editor-in-Chief of *Mechanics of Advanced Materials and Structures*, *International Journal of Computational Methods in Engineering Science and Mechanics*, and *International Journal of Structural Stability and Dynamics*; he also serves on the editorial boards of over two dozen other journals, including *International Journal for Numerical Methods in Engineering, Computer Methods in Applied Mechanics and Engineering,* and *International Journal of Non-Linear Mechanics.*

Dr. Reddy is one of the selective researchers in engineering around world who is recognized by ISI Highly Cited Researchers with over 10,000 citations with H-index of over 50. A more complete resume with links to journal papers can be found at (this web site lists only highly cited researchers in engineering around the world): http://isihighlycited.com/ or http://www.tamu.edu/acml.