

# Introduction to Continuum Mechanics

covering linearized elasticity, fluid mechanics, heat  
transfer, and viscoelasticity

**Second Edition**



# Introduction to Continuum Mechanics

covering linearized elasticity, fluid mechanics, heat  
transfer, and viscoelasticity

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## Preface to the Second Edition

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Engineers are problem solvers. They construct mathematical models, develop analysis approaches and methodologies, and design and manufacture various types of systems or processes. Mathematical development and engineering analysis are aids to designing systems for specific functionality, and they involve (1) mathematical model development, (2) data acquisition by measurements, (3) numerical simulation, and (4) evaluation of the results in light of known information. Mathematical models are developed using laws of physics and assumptions concerning the system behavior. The most difficult step in arriving at a design that is both functional and cost-effective is the construct of a suitable mathematical model of the system behavior. It is in this context a course on continuum mechanics or elasticity provides engineers with a background to formulate a suitable mathematical model and evaluate it in the context of the functionality and design constraints placed on the system.

The second edition of *Introduction to Continuum Mechanics* has the same objective as the first one, namely, to facilitate an easy and thorough understanding of concepts from a first course on continuum mechanics and elasticity. The course also helps engineers who depend on canned programs to analyze problems in interpreting the results produced by such programs. **The book offers a concise but rigorous treatment of the subject of continuum mechanics and elasticity at the introductory level.** In all of the chapters of the second edition, additional explanations, examples, and problems are added. No attempt is made to enlarge the scope or increase coverage of topics.

The book may be used as a text book for a first course on continuum mechanics as well as elasticity (omitting Chapter 8 on fluid mechanics and heat transfer). A solutions manual is also prepared for the book. The solution manual is made available by the publisher only to those instructors who adopt the book as a textbook for a course. Since the publication of the first edition several users of the book communicated their compliments as well as errors they found, and the author is grateful to them. All of the errors known to the author have been corrected in the current edition. The author is grateful, in particular, to Professors Karan Surana (University of Kansas), Arun Srinivasa (Texas A&M University), Srikanth Vedantam (Indian Institute of Technology, Madras), and Rebecca Brannon (University of Utah) for their constructive comments and advice.

*J. N. Reddy*

*Tis the good reader that makes the good book; in every book he finds passages which seem confidences or asides hidden from all else and unmistakeably meant for his ear; the profit of books is according to the sensibility of the reader; the profoundest thought or passion sleeps as in a mine, until it is discovered by an equal mind and heart.*

Ralph Waldo Emerson

*You cannot teach a man anything, you can only help him find it within himself.*

Galileo Galilei



## Preface (to the First Edition)

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*If I have been able to see further, it was only because I stood on the shoulders of giants.*

Isaac Newton

Many of the mathematical models of natural phenomena are based on fundamental scientific laws of physics or otherwise, extracted from centuries of research on the behavior of physical systems under the action of natural "forces". Today this subject is referred to simply as *mechanics* - a phrase that encompasses broad fields of science concerned with the behavior of fluids, solids, and complex materials. Mechanics is vitally important to virtually every area of technology and remains an intellectually rich subject taught in all major universities. It is also the focus of research in departments of aerospace, chemical, civil, and mechanical engineering, and engineering science and mechanics, as well as applied mathematics and physics. The last several decades have witnessed a great deal of research in continuum mechanics and its application to a variety of problems. As most modern technologies are no longer discipline-specific but involve multidisciplinary approaches, scientists and engineers should be trained to think and work in such environments. Therefore, it is necessary to introduce the subject of mechanics to senior undergraduate and beginning graduate students so that they have a strong background in the basic principles common to all major engineering fields. A first course on *continuum mechanics* or *elasticity* is the one that provides the basic principles of mechanics and prepares engineers and scientists for advanced courses in traditional as well as emerging fields such as biomechanics and nanomechanics.

There are many books on mechanics of continua. These books fall into two major categories: those which present the subject as highly mathematical and abstract subject, and those which are too elementary to be of use for those who will pursue further work in fluid dynamics, elasticity, plates and shells, viscoelasticity, plasticity, and interdisciplinary areas such as geomechanics, biomechanics, mechanobiology, and nanoscience. As is the case with all other books written (solely) by the author, the objective is to facilitate an easy understanding of the topics covered. While the author is fully aware that he is not an authority on the subject of this book, he feels that he understands the concepts well and feels confident to explain to others. It is hoped that the book, which is simple in presenting the main concepts yet mathematically rigorous enough in providing the invariant form as well as component form of the governing equations for analysis of practical problems of engineering. In particular, the book contains formulations and applications to specific problems from heat transfer, fluid mechanics, and solid mechanics.

The motivation and encouragement that led to the writing of this book came from the experience of teaching a course on continuum mechanics at Virginia Polytechnic Institute and State University and Texas A&M University. A course

on continuum mechanics takes different forms - abstract to very applied - when taught by different people. The primary objective of the course taught by the author is two-fold: (1) formulation of equations that describe the motion and thermomechanical response of materials, and (2) solution of these equations for specific problems from elasticity, fluid flows, and heat transfer. The present book is a formal presentation of the author’s notes developed for such a course over last two and half decades.

With a brief discussion of the concept of a continuum in Chapter 1, a review of vectors and tensors is presented in Chapter 2. Since the language of mechanics is mathematics, it is necessary for all readers to familiarize themselves with the notation and operations of vectors and tensors. The subject of kinematics is discussed in Chapter 3. Various measures of strain are introduced here. The deformation gradient, Cauchy–Green deformation, Green–Lagrange strain, Cauchy and Euler strain, rate of deformation, and vorticity tensors are introduced and the polar decomposition theorem is discussed in this chapter. In Chapter 4, various measures of stress - Cauchy stress and Piola–Kirchhoff stress measures - are introduced, and stress equilibrium equations are presented.

Chapter 5 is dedicated to the derivation of the field equations of continuum mechanics, which forms the heart of the book. The field equations are derived using the principles of conservation of mass, momenta, and energy. Constitutive relations that connect the kinematic variables (e.g., density, temperature, deformation) to the kinetic variables (e.g., internal energy, heat flux, and stresses) are discussed in Chapter 6 for elastic materials, viscous and viscoelastic fluids, and heat transfer.

Chapters 7 and 8 are devoted to the application of the field equations derived in Chapter 5 and constitutive models of Chapter 6 to problems of linearized elasticity, and fluid mechanics and heat transfer, respectively. Simple boundary-value problems, mostly linear, are formulated and their solutions are discussed. The material presented in these chapters illustrate how physical problems are analytically formulated with the aid of continuum equations. Chapter 9 deals with linear viscoelastic constitutive models and their application to simple problems of solid mechanics. Since a continuum mechanics course is mostly offered by solid mechanics programs, the coverage in this book is slightly more favorable, in terms of the amount and type of material covered, to solid and structural mechanics.

The book is written keeping the undergraduate seniors and first-year graduate students of engineering in mind. Therefore, it is most suitable as a text book for adoption for a first course on continuum mechanics or elasticity. The book also serves as an excellent precursor to courses on viscoelasticity, plasticity, nonlinear elasticity, and nonlinear continuum mechanics.

The book contains so many mathematical equations that it is hardly possible not to have typographical and other kinds of errors. I wish to thank in advance those readers who are willing to draw the author’s attention to typos and errors, using the e-mail address: *jnreddytamu.edu*.

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# Contents

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Preface (to the second edition) . . . . .	xvii
Preface (to the first edition) . . . . .	xix
<b>1 INTRODUCTION . . . . .</b>	<b>1</b>
1.1 Continuum Mechanics . . . . .	1
1.2 A Look Forward . . . . .	4
1.3 Summary . . . . .	5
Problems . . . . .	6
<b>2 VECTORS AND TENSORS . . . . .</b>	<b>9</b>
2.1 Background and Overview . . . . .	9
2.2 Vector Algebra . . . . .	10
2.2.1 Definition of a Vector . . . . .	10
2.2.1.1 Vector addition . . . . .	11
2.2.1.2 Multiplication of vector by scalar . . . . .	11
2.2.1.3 Linear independence of vectors . . . . .	11
2.2.2 Scalar and Vector Products . . . . .	12
2.2.2.1 Scalar product . . . . .	12
2.2.2.2 Vector product . . . . .	13
2.2.2.3 Triple product of vectors . . . . .	16
2.2.3 Plane Area as a Vector . . . . .	17
2.2.4 Reciprocal Basis . . . . .	19
2.2.4.1 Components of a vector . . . . .	19
2.2.4.2 General basis . . . . .	19
2.2.4.3 Orthonormal basis . . . . .	21
2.2.4.4 The Gram–Schmidt orthonormalization . . . . .	22
2.2.5 Summation Convention . . . . .	23
2.2.5.1 Dummy index . . . . .	24
2.2.5.2 Free index . . . . .	24
2.2.5.3 Kronecker delta . . . . .	25
2.2.5.4 Permutation symbol . . . . .	25
2.2.6 Transformation Law for Different Bases . . . . .	28
2.2.6.1 General transformation laws . . . . .	28
2.2.6.2 Transformation laws for orthonormal systems . . . . .	29

x

2.3	Theory of Matrices . . . . .	31
2.3.1	Definition . . . . .	31
2.3.2	Matrix Addition and Multiplication of a Matrix by a Scalar . . . . .	32
2.3.3	Matrix Transpose . . . . .	33
2.3.4	Symmetric and Skew Symmetric Matrices . . . . .	33
2.3.5	Matrix Multiplication . . . . .	34
2.3.6	Inverse and Determinant of a Matrix . . . . .	36
2.3.6	Positive-Definite and Orthogonal Matrices . . . . .	39
2.4	Vector Calculus . . . . .	40
2.4.1	Differentiation of a Vector with Respect to a Scalar . . . . .	40
2.4.2	Curvilinear Coordinates . . . . .	42
2.4.3	The Fundamental Metric . . . . .	43
2.4.4	Derivative of a Scalar Function of a Vector . . . . .	44
2.4.5	The Del Operator . . . . .	45
2.4.6	Divergence and Curl of a Vector . . . . .	47
2.4.7	Cylindrical and Spherical Coordinate Systems . . . . .	51
2.4.8	Gradient, Divergence, and Curl Theorems . . . . .	52
2.5	Tensors . . . . .	53
2.5.1	Dyads and Dyadics . . . . .	53
2.5.2	Nonion Form of a Second-Order Tensor . . . . .	54
2.5.3	Transformation of Components of a Tensor . . . . .	57
2.5.4	Higher-Order Tensors . . . . .	58
2.5.5	Tensor Calculus . . . . .	59
2.5.6	Eigenvalues and Eigenvectors of Tensors . . . . .	62
2.5.6.1	Eigenvalue problem . . . . .	62
2.5.6.2	Eigenvalues and eigenvectors of a real symmetric matrix . . . . .	62
2.5.6.3	Spectral theorem . . . . .	64
2.5.6.4	Calculation of eigenvalues and eigenvectors . . . . .	64
2.6	Chapter Summary . . . . .	72
	Problems . . . . .	73
<b>3</b>	<b>KINEMATICS OF CONTINUA . . . . .</b>	<b>81</b>
3.1	Introduction . . . . .	81
3.2	Descriptions of Motion . . . . .	82
3.2.1	Configurations of a Continuous Medium . . . . .	82

<i>CONTENTS</i>	<b>xi</b>
3.2.2 Material Description . . . . .	83
3.2.3 Spatial Description . . . . .	85
3.2.4 Displacement Field . . . . .	88
3.3 Analysis of Deformation . . . . .	89
3.3.1 Deformation Gradient . . . . .	89
3.3.2 Isochoric, Homogeneous, and Inhomogeneous Deformations . .	93
3.3.2.1 Isochoric deformation . . . . .	93
3.3.2.2 Homogeneous deformation . . . . .	93
3.3.2.3 Nonhomogeneous deformation . . . . .	95
3.3.3 Change of Volume and Surface . . . . .	96
3.3.3.1 Volume change . . . . .	96
3.3.3.2 Area change . . . . .	97
3.4 Strain Measures . . . . .	98
3.4.1 Cauchy–Green Deformation Tensors . . . . .	98
3.4.2 Green–Lagrange Strain Tensor . . . . .	100
3.4.3 Physical Interpretation of Green–Lagrange Strain Components	101
3.4.4 Cauchy and Euler Strain Tensors . . . . .	103
3.4.5 Transformation of Strain Componets . . . . .	106
3.4.6 Invariants and Principal Values of Strains . . . . .	109
3.5 Infinitesimal Strain Tensor and Rotation Tensor . . . . .	111
3.5.1 Infinitesimal Strain Tensor . . . . .	111
3.5.2 Physical Interpretation of Infinitesimal Strain Tensor Components . . . . .	112
3.5.3 Infinitesimal Rotation Tensor . . . . .	114
3.5.4 Infinitesimal Strains in Cylindrical and Spherical Coordinate Systems . . . . .	116
3.5.4.1 Cylindrical coordinate system . . . . .	117
3.5.4.2 Spherical coordinate system . . . . .	117
3.6 Velocity Gradient and Vorticity Tensors . . . . .	118
3.6.1 Definitions . . . . .	118
3.6.2 Relationship Between $\mathbf{D}$ and $\dot{\mathbf{E}}$ . . . . .	119
3.7 Compatibility Equations . . . . .	120
3.7.1 Preliminary Comments . . . . .	120
3.7.2 Infinitesimal Strains . . . . .	121

3.7.3	Finite Strains . . . . .	125
3.8	Rigid Body Motions and Material Objectivity . . . . .	125
3.8.1	Preliminary Comments . . . . .	125
3.8.1.1	Introduction and rigid body transformation . . . . .	125
3.8.1.2	Effect on $\mathbf{F}$ . . . . .	128
3.8.1.3	Effect on $\mathbf{C}$ and $\mathbf{E}$ . . . . .	128
3.8.1.4	Effect on $\mathbf{L}$ and $\mathbf{D}$ . . . . .	129
3.8.2	Material Objectivity . . . . .	129
3.8.2.1	Observer transformation . . . . .	129
3.8.2.2	Objectivity of various kinematic measures . . . . .	130
3.8.2.3	Time rate of change in a rotating frame of reference . . . . .	131
3.9	Polar Decomposition Theorem . . . . .	132
3.9.1	Preliminary Comments . . . . .	132
3.9.2	Rotation and Stretch Tensors . . . . .	132
3.9.3	Objectivity of Stretch Tensors . . . . .	138
3.10	Chapter Summary . . . . .	139
Problems	. . . . .	140
<b>4</b>	<b>STRESS MEASURES . . . . .</b>	<b>151</b>
4.1	Introduction . . . . .	151
4.2	Cauchy Stress Tensor and Cauchy’s Formula . . . . .	151
4.2.1	Stress Vector . . . . .	151
4.2.2	Cauchy’s Formula . . . . .	152
4.2.3	Cauchy Stress Tensor . . . . .	153
4.3	Transformation of Stress Components and Principal Stresses . . . . .	157
4.3.1	Transformation of Stress Components . . . . .	157
4.3.1.1	Invariants . . . . .	157
4.3.1.2	Transformation equations . . . . .	157
4.3.2	Principal Stresses and Principal Planes . . . . .	160
4.3.3	Maximum Shear Stress . . . . .	162
4.4	Other Stress Measures . . . . .	164
4.4.1	Preliminary Comments . . . . .	164
4.4.2	First Piola–Kirchhoff Stress Tensor . . . . .	164
4.4.3	Second Piola–Kirchhoff Stress Tensor . . . . .	165

4.5	Equilibrium Equations for Small Deformations . . . . .	169
4.6	Objectivity of Stress Tensors . . . . .	172
4.6.1	Cauchy Stress Tensor . . . . .	172
4.6.2	First Piola–Kirchhoff Stress Tensor . . . . .	172
4.6.3	Second Piola–Kirchhoff Stress Tensor . . . . .	172
4.7	Chapter Summary . . . . .	173
Problems	. . . . .	174
<b>5</b>	<b>CONSERVATION AND BALANCE LAWS . . . . .</b>	<b>181</b>
5.1	Introduction . . . . .	181
5.2	Conservation of Mass . . . . .	182
5.2.1	Preliminary Discussion . . . . .	182
5.2.2	Material Time Derivative . . . . .	182
5.2.3	Vector and Integral Identities . . . . .	184
5.2.3.1	Vector identities . . . . .	184
5.2.3.2	Integral identities . . . . .	185
5.2.4	Continuity Equation in the Spatial Description . . . . .	185
5.2.5	Continuity Equation in the Material Description . . . . .	191
5.2.6	Reynolds Transport Theorem . . . . .	193
5.3	Conservation of Momenta . . . . .	193
5.3.1	Principle of Conservation of Linear Momentum . . . . .	193
5.3.1.1	Equations of motion in the spatial description . . . . .	197
5.3.1.2	Equations of motion in the material description . . . . .	199
5.3.2	Spatial Equations of Motion in Cylindrical and Spherical Coordinates . . . . .	201
5.3.2.1	Cylindrical coordinates . . . . .	202
5.3.2.2	Spherical coordinates . . . . .	202
5.3.3	Principle of Balance of Angular Momentum . . . . .	203
5.3.3.1	Monopolar case . . . . .	203
5.3.3.2	Multipolar case . . . . .	205
5.4	Thermodynamic Principles . . . . .	206
5.4.1	Introduction . . . . .	206
5.4.2	Balance of Energy . . . . .	207
5.4.2.1	Energy equation in the spatial description . . . . .	207
5.4.2.2	Energy equation in the material description . . . . .	209

xiv

5.4.3 Entropy Inequality . . . . .	210
5.4.3.1 Homogeneous processes . . . . .	210
5.4.3.2 Inhomogeneous processes . . . . .	210
5.5 Chapter Summary . . . . .	212
5.5.1 Preliminary Comments . . . . .	212
5.5.2 Conservation and Balance Equations in the Spatial Description	212
5.5.3 Conservation and Balance Equations in the Material Description	213
5.5.4 Closing Comments . . . . .	213
Problems . . . . .	214
<b>6 CONSTITUTIVE EQUATIONS. . . . .</b>	<b>221</b>
6.1 Introduction . . . . .	221
6.1.1 General Comments . . . . .	221
6.1.2 General Principles of Constitutive Theory . . . . .	222
6.1.3 Material Frame Indifference . . . . .	223
6.1.4 Restrictions Placed by the Entropy Inequality . . . . .	224
6.2 Elastic Materials . . . . .	225
6.2.1 Cauchy Elastic Materials. . . . .	225
6.2.2 Green-Elastic or Hyperelastic Materials . . . . .	226
6.2.3 Linearized Hyperelastic Materials . . . . .	227
6.3 Hookean Solids . . . . .	228
6.3.1 Generalized Hooke’s Law . . . . .	228
6.3.2 Material Symmetry Planes . . . . .	230
6.3.3 Monoclinic Materials . . . . .	232
6.3.4 Orthotropic Materials . . . . .	233
6.3.5 Isotropic Materials . . . . .	237
6.4 Nonlinear Elastic Constitutive Relations . . . . .	241
6.5 Newtonian Fluids . . . . .	242
6.5.1 Introduction . . . . .	242
6.5.2 Ideal Fluids . . . . .	243
6.5.3 Viscous Incompressible Fluids . . . . .	243
6.6 Generalized Newtonian Fluids . . . . .	245
6.6.1 Introduction . . . . .	245

6.6.2 Inelastic fluids . . . . .	245
6.6.2.1 Power-law model . . . . .	246
6.6.2.2 Carreau model . . . . .	246
6.6.2.3 Bingham model . . . . .	247
6.6.3 Viscoelastic Constitutive Models . . . . .	247
6.6.3.1 Differential models . . . . .	247
6.6.3.2 Integral models . . . . .	250
6.7 Heat Transfer . . . . .	251
6.7.1 Introduction . . . . .	251
6.7.2 Fourier’s Heat Conduction Law . . . . .	251
6.7.3 Newton’s Law of Cooling . . . . .	252
6.7.4 Stefan–Boltzmann Law . . . . .	252
6.8 Constitutive Relations for Coupled Problems . . . . .	252
6.8.1 Electromagnetics . . . . .	252
6.8.1.1 Maxwell’s equations . . . . .	253
6.8.1.2 Constitutive relations . . . . .	253
6.8.2 Thermoelasticity . . . . .	255
6.8.3 Hygrothermal elasticity . . . . .	255
6.8.4 Electroelasticity . . . . .	256
6.9 Chapter Summary . . . . .	258
Problems . . . . .	259
<b>7 LINEARIZED ELASTICITY . . . . .</b>	<b>265</b>
7.1 Introduction . . . . .	265
7.2 Governing Equations . . . . .	265
7.2.1 Preliminary Comments . . . . .	266
7.2.2 Summary of Equations . . . . .	266
7.2.2.1 Strain-displacement equations . . . . .	266
7.2.2.2 Equations of motion . . . . .	267
7.2.2.3 Constitutive equations . . . . .	268
7.2.2.4 Boundary conditions . . . . .	269
7.2.2.5 Compatibility conditions . . . . .	269
7.2.3 The Navier Equations . . . . .	269
7.2.4 The Beltrami–Michell Equations . . . . .	270
7.3 Solution Methods . . . . .	271

7.3.1	Types of Problems . . . . .	271
7.3.2	Types of Solution Methods . . . . .	272
7.3.3	Examples of Semi-Inverse Method . . . . .	273
7.3.4	Stretching and Bending of Beams . . . . .	278
7.3.5	Superposition Principle . . . . .	283
7.3.6	Uniqueness of Solutions . . . . .	284
7.4	Clapeyron’s, Betti’s, and Maxwell’s Theorems . . . . .	285
7.4.1	Clapeyron’s Theorem . . . . .	285
7.4.2	Betti’s Reciprocity Theorem . . . . .	288
7.4.3	Maxwell’s Reciprocity Theorem . . . . .	291
7.5	Solution of Two-Dimensional Problems . . . . .	293
7.5.1	Introduction . . . . .	293
7.5.2	Plane Strain Problems . . . . .	294
7.5.3	Plane Stress Problems . . . . .	297
7.5.4	Unification of Plane Stress and Plane Strain Problems . . . . .	300
7.5.5	Airy Stress Function . . . . .	301
7.5.6	Saint–Venant’s Principle . . . . .	303
7.5.7	Torsion of Cylindrical Members . . . . .	308
7.5.7.1	Warping function . . . . .	309
7.5.7.2	Prandtl’s stress function . . . . .	311
7.6	Methods Based on Total Potential Energy . . . . .	314
7.6.1	Introduction . . . . .	314
7.6.2	The Variational Operator . . . . .	314
7.6.3	The Principle of the Minimum Total Potential Energy . . . . .	316
7.6.3.1	Construction of the total potential energy functional . . . . .	316
7.6.3.2	Euler’s equations and natural boundary conditions . . . . .	317
7.6.3.3	The minimum property of the total potential energy functional . . . . .	319
7.6.4	Castigliano’s Theorem I . . . . .	322
7.6.5	The Ritz Method . . . . .	326
7.6.5.1	The variational problem . . . . .	326
7.6.5.2	Description of the method . . . . .	328
7.7	Hamilton’s Principle . . . . .	334
7.7.1	Introduction . . . . .	334
7.7.2	Hamilton’s Principle for a Rigid Body . . . . .	334



7.7.3 Hamilton’s Principle for a Continuum . . . . .	338
7.8 Chapter Summary . . . . .	341
Problems . . . . .	342
<b>8 FLUID MECHANICS AND HEAT TRANSFER . . . . .</b>	<b>355</b>
8.1 Governing Equations . . . . .	355
8.1.1 Preliminary Comments . . . . .	355
8.1.2 Summary of Equations . . . . .	356
8.2 Fluid Mechanics Problems . . . . .	357
8.2.1 Governing Equations of Viscous Flows . . . . .	357
8.2.2 Inviscid Fluid Statics . . . . .	360
8.2.3 Parallel Flow (Navier-Stokes Equations) . . . . .	362
8.2.4 Problems with Negligible Convective Terms . . . . .	367
8.2.5 Energy Equation for One-Dimensional Flows . . . . .	370
8.3 Heat Transfer Problems . . . . .	373
8.3.1 Governing Equations . . . . .	373
8.3.2 Heat Conduction in a Cooling Fin . . . . .	374
8.3.3 Axisymmetric Heat Conduction in a Circular Cylinder . . . . .	376
8.3.4 Two-Dimensional Heat Transfer . . . . .	379
8.3.5 Coupled Fluid Flow and Heat Transfer . . . . .	381
8.4 Chapter Summary . . . . .	382
Problems . . . . .	382
<b>9 LINEARIZED VISCOELASTICITY . . . . .</b>	<b>387</b>
9.1 Introduction . . . . .	387
9.1.1 Preliminary Comments . . . . .	387
9.1.2 Initial Value Problem, The Unit Impulse, and the Unit Step Function . . . . .	388
9.1.3 The Laplace Transform Method . . . . .	390
9.2 Spring and Dashpot Models . . . . .	393
9.2.1 Creep Compliance and Relaxation Modulus . . . . .	393
9.2.2 Maxwell Element . . . . .	395
9.2.2.1 Creep response . . . . .	396
9.2.2.2 Relaxation response . . . . .	397

xviii

9.2.3 Kelvin–Voigt Element . . . . .	397
9.2.3.1 Creep response . . . . .	398
9.2.3.2 Relaxation response . . . . .	398
9.2.4 Three-Element Models . . . . .	399
9.2.5 Four-Element Models . . . . .	401
9.3 Integral Constitutive Equations . . . . .	405
9.3.1 Hereditary Integrals . . . . .	405
9.3.2 Hereditary Integrals for Deviatoric Components . . . . .	408
9.3.3 The Correspondence Principle . . . . .	409
9.3.4 Elastic and Viscoelastic Analogies . . . . .	412
9.4 Summary . . . . .	417
Problems . . . . .	417
<b>REFERENCES . . . . .</b>	<b>421</b>
<b>ANSWERS TO SELECTED PROBLEMS . . . . .</b>	<b>425</b>
<b>SUBJECT INDEX . . . . .</b>	<b>437</b>

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## About the Author

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